QUADRATIC EQUATIONS

There are three usual methods for solving quadratics

1. Factorising
2. Quadratic Formula
3. Completing the Square

We call the answers to a quadratic equation the roots of the equation

FACTORISING

Exercise 1.

Find the solutions to the equations:

(i) $(x+2)(x-1)=0$
(ii) $(x-3)(x+1)=0$
(iii) $(x+4)(x-2)=0$
(iv) $x(x-2)=0$
(v) $x(x+3)=0$
(vi) $2x(x-3)=0$
(vii) $(2x-1)(2x-5)=0$
(viii) $(2x-9)(2x+3)=0$
(ix) $(x-6)^2=0$
(x) $(2x-7)^2=0$

Exercise 2.

Factorise and solve the equations:

(i) $x^2-2x=0$
(ii) $2x^2+x=0$
(iii) $5x^2-15x=0$
(iv) $2x^2+4x=0$
(v) $x^2+\frac{1}{2}x=0$
(vi) $x^2-16=0$
(vii) $9x^2-16=0$
(viii) $7x^2-175=0$
(ix) $x^2-{}^1/_{25}=0$
(x) $9x^2-{}^1/_9=0$
(xi) $16x^2-{}^1/_{64}=0$
(xii) $x^2-5x+6=0$
(xiii) $x^2-2x-15=0$
(xiv) $x^2+8x-33=0$
(xv) $2x^2+3x+1=0$
(xvi) $3x^2-4x+1=0$
(xvii) $7x^2+x-8=0$
(xviii) $2x^2+x-15=0$
(xix) $2x^2+16x-66=0$
(xx) $3x^2+16x-12=0$

Exercise 3.

Tidy up, factorise and solve the following:

(i) $x(x+24)=0$
(ii) $x(2x+7)+6=0$
(iii) $(x+3)(x+5)=3+x$
(iv) $(2x-3)(x+4)=3+4x$
(v) $2x-2(x-1)=x(x-1)$
(vi) $2x(x-4)+4x+5=3(x+3)$
(vii) $5(2x^2-3x-2)=3(x^2-6x-2)$
(viii) $(x+5)(x+6)=6$
(ix) Solve $x^2-6x+5=0$. Hence, find the values of t for which $(t-2)^2-6(t-2)+5=0$.
(x) Solve $x^2-29x+100=0$. Hence, solve $y^4-29y^2+100=0$.

QUADRATIC FORMULA

The Quadratic Formula uses the "a", "b", and "c" from "$ax^2 + bx + c$", where "a", "b", and "c" are just numbers:

For $ax^2 + bx + c = 0$, the values of x are given by: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Exercise 4.

Write each of the following in their simplest form: - *"Show your work first and then test using your calculator"*

$\sqrt{18}, \sqrt{48}, \sqrt{72}, \sqrt{98}, \sqrt{45}, \sqrt{80}, \sqrt{125}$

Exercise 5.

Use the quadratic formula to solve the following. *"Give your answers in their simplest form"*

(i) $x^2 - 4x + 2 = 0$
(ii) $x^2 - 2x - 7 = 0$
(iii) $x^2 - 8x - 1 = 0$
(iv) $x^2 - 6x + 4 = 0$
(v) $x^2 + 2x - 1 = 0$
(vi) $x^2 + 4x + 1 = 0$
(vii) $4x^2 - 4x - 7 = 0$
(viii) $2x^2 - 2x - 1 = 0$
(ix) $4x^2 - 8x + 1 = 0$
(x) $x^2 - 6x + 1 = 0$
(xi) $x^2 - 4x = 0$
(xii) $3x^2 = 5x$
(xiii) $5 = x(3x + 14)$
(xiv) $18 + 5x^2 = 33x$
(xv) $21x^2 + 22x + 5 = 0$
(xvi) $(x + 4)(x - 1) = -5$
(xvii) $(x + 6)(x + 2) = 13$

Exercise 6.

Tidy up the following and solve for the variable, x :-

Method:

1. Factorise the denominator, if needed.
2. Multiply all terms by the common denominator
3. Multiply out any brackets
4. Tidy up!
5. Solve the quadratic

(i) $x^2 = \dfrac{3x+5}{6}$
(ii) $\dfrac{3}{x} + \dfrac{5}{x+2} = 2$
(iii) $\dfrac{1}{x} = 2 - \dfrac{2}{x+1}$
(iv) $\dfrac{9}{20} - \dfrac{1}{x} = \dfrac{1}{x+1}$
(v) $\dfrac{x+10}{x-5} - \dfrac{11}{6} = \dfrac{10}{x}$
(vi) $\dfrac{4}{(x+6)(x+4)} - 7 = \dfrac{2}{x+4}$
(vii) $\dfrac{1}{4x^2-1} - \dfrac{x}{2x+1} = 0$
(viii) $\dfrac{x(x-7)}{x-5} + \dfrac{10}{x-5} = \dfrac{x}{2}$
(ix) $\dfrac{1}{16x^2+8x-3} + \dfrac{1}{12} + \dfrac{1}{4(4x+3)} = 0$
(x) $\dfrac{6-4x}{x^2-9} + \dfrac{3}{x+3} + \dfrac{2x}{x-3} = 0$

COMPLETING THE SQUARE

Exercise 7.

Which of the following are Perfect squares: (*Place a ✓ or an ✗*)

(i) $x^2 - 8x + 16$
(ii) $x^2 + 4x + 4$
(iii) $x^2 - 10x + 25$
(iv) $x^2 + 20x + 100$
(v) $x^2 - 16x + 64$
(vi) $x^2 - 7x + 49$
(vii) $x^2 + 14x + 49$
(viii) $x^2 - 12x + 36$
(ix) $x^2 + 11x + 91$
(x) $x^2 - 3x + {}^9/_4$
(xi) $x^2 - 9x + {}^{81}/_4$
(xii) $x^2 + 10x - 2$

Exercise 8.

By completing the square, solve the following quadratic equations

(i) $x^2 - 4x - 10 = 0$
(ii) $x^2 + 6x - 20 = 0$
(iii) $x^2 - 5x + 2 = 0$
(iv) $x^2 + 7x - 3 = 0$
(v) $2x^2 - 5x - 7 = 0$
(vi) $3x^2 - 4x - 11 = 0$
(vii) $3x^2 + 5x - 12 = 0$
(viii) $4x^2 + 5x - 13 = 0$
(ix) $x^2 - 4x + 7 = 0$
(x) $x^2 - 8x + 7 = 0$
(xi) $x^2 - 10x - 2 = 0$
(xii) $x^2 + 10x + 13 = 0$

Word Equations

Exercise 9

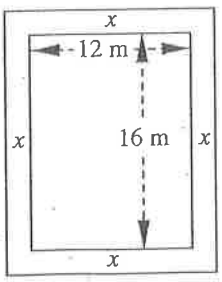
of the garden.

2. A window has a height that is $\frac{4}{3}$ times longer than its width. If the area of the window is 192 m^2, what are the dimensions of the window?

3. The length of a rectangle is 8 cm less than twice its width. If the area of the rectangle is 120 cm^2, what is the length of the diagonal, leaving your answer in surd form?

4. In a right-angled triangle, the length of the hypotenuse is 10 cm. Of the two shorter sides, one side is 2 cm longer than the other side.
   (i) Find the lengths of the two shorter sides.
   (ii) Hence, find the area of the triangle.

5. A garden measuring 12 m by 16 m is to have a pedestrian pathway installed all around it, increasing the total area to 285 m^2.
   What will be the width of the pathway?



6. A rectangular garden is 30 m long and 20 m wide. A path of uniform width is set around the edge that reduces its area to 375 m^2. What is the width of the path?

7. A rectangular garden measures 20 m by 30 m. A pathway is laid around the garden that reduces its area by 264 m^2. How wide is the pathway?

8. The sum of the areas of two circles is 106π cm^2 and the radius of the larger circle is 4 cm longer than the radius of the smaller circle. Find the lengths of the radii.

9. 132 chocolates are equally divided among a number of people at a party. If the number of chocolates that each receives is one more than the number of people, find how many people were at the party.

10. You have to make an open square-bottomed box with a height of 3 cm and a volume of 75 cm^3. To do this, you will take a square piece of cardboard, cutting 3 cm squares from each corner, scoring between the corners and folding up the edges. Find the dimensions of the cardboard.

11. A rectangular piece of cardboard is 2 cm longer than it is wide. From each of its corners, a square piece 2 cm on a side is cut out. The flaps are then turned up to form an open box that has a volume of 70 cm^3. Calculate the dimensions of the cardboard.

13. The perimeter of a triangle is $\frac{17x}{24}$ and the lengths of two of the sides are $\frac{3x}{8}$ and $\frac{2x-5}{12}$.
   Find the length of the third side in terms of x.

14. The perimeter of a rectangle is $\frac{14x}{15}$ and the measure of each length is $\frac{x+2}{3}$.
   Find the measure of each width in terms of x.

15. The numerator of a fraction is 8 less than the denominator of the fraction.
   (i) If the numerator is x, write the denominator in terms of x.
   (ii) If this fraction can be broken down to be $\frac{3}{5}$, write an equation to represent this information and hence solve for x.
   (iii) Write down the fraction.

The total cost of entry for the students to

Quadratic Graphs

Exercise 10

Find the coordinates of the points where the quadratic function crosses the x and y axis and hence sketch the graph:

(i) $x^2 + 7x + 10 = y$
(ii) $x^2 + 3x - 4 = y$
(iii) $2x^2 - 18 = y$
(iv) $-x^2 + 7x - 10 = y$
(v) $-3x^2 - 7x + 10 = y$
(vi) $2x^2 + 3x - 14 = y$
(vii) $x^2 - 8x + 16 = y$
(viii) $-x^2 + 9x - 14 = y$
(ix) $-2x^2 + 6x + 8 = y$

Exercise 11

Construct a quadratic equation given the following roots:

(i) -2, 5 (ii) -3, -4 (iii) $\frac{1}{2}$, 3 (iv) $\frac{1}{3}, -\frac{1}{2}$ (v) -4, 3 (vi) $\frac{2}{3}, -\frac{1}{3}$

Exercise 12

11. Prove that the roots of the equation $x^2 - 2px + (p^2 - q^2) = 0$ are real for all $p, q \in \mathbb{R}$.

12. Prove that the roots of the equation $x^2 - (a + d)x - (c + b)^2 = 0$ are real for all $a, b, c, d \in \mathbb{R}$.

13. Prove that the roots of the equation $kx^2 + (3 - 2k^2)x - 6k = 0$ are real for all $k \in \mathbb{R}$. Express these roots in terms of k.

14. Prove that the roots of the equation
$$px^2 - (p + q)x + q = 0$$
are real for all $p, q \in \mathbb{R}$. Express these roots in terms of p and q.

15. Show that the roots of the equation
$$px^2 + (2p - q)x - 2q = 0$$
are real for all $p, q \in \mathbb{R}$. Express these roots in terms of p and q.

16. If the roots of the equation
$$x^2 - 2ax + b - 4a^2 = 0$$
are equal, express b in terms of a.

17. Determine the values of $k \in \mathbb{R}$ if the roots of the equation
$$(3k + 4)x^2 - (3k - 1)x + 4 = 0$$
are equal.

18. (i) Solve the equation $x^2 - 8x + 12 = 0$.
(ii) Hence, find the solutions of the equation $(x^2 + x)^2 - 8(x^2 + x) + 12 = 0$.

19. (i) Solve the equation $2x^2 - 5x - 12 = 0$.
(ii) Hence, find the solutions of the equation $2(3t + 1)^2 - 5(3t + 1) - 12 = 0$.

20. (i) Solve the equation $x^2 - 13x + 36 = 0$.
(ii) Hence, find the solutions of the equation $x^4 - 13x^2 + 36 = 0$.

21. (i) Solve the equation $x^2 - 18x + 72 = 0$.
(ii) Hence, find the solutions of the equation $(x^2 + x)^2 + 72 = 18(x^2 + x)$.