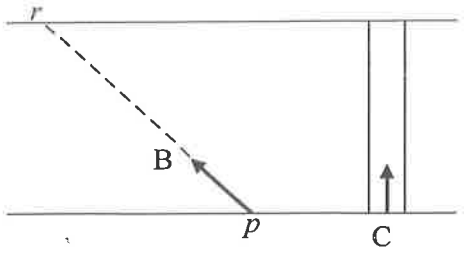
Rel Vel

2. A river is 72 metres wide and has parallel banks.
A boat B departs from point p on the southern bank and lands at point r on the northern bank.



2007 (o)

The actual velocity of B is $-4\vec{i}+3\vec{j}$ m/s.
Cyclist C travels due north at a constant speed of 4 m/s across a straight level bridge which spans the river.

Find

(i) the velocity of C in terms of $\vec{i}$ and $\vec{j}$
(ii) the velocity of B relative to C in terms of $\vec{i}$ and $\vec{j}$
(iii) the magnitude and direction of the velocity of B relative to C
(iv) the time it takes C to cross the river
(v) how much longer it will take B to cross the river.

(i) $\vec{V}_C = 0\,\vec{i}+4\,\vec{j}$ — 10

(ii) $\vec{V}_{BC} = \vec{V}_B - \vec{V}_C$ — 5
$= (-4\,\vec{i}+3\,\vec{j})-(0\,\vec{i}+4\,\vec{j})$
$= -4\,\vec{i}-1\,\vec{j}$ — 5

(iii) magnitude $= \sqrt{(-4)^2+(-1)^2}$
$= \sqrt{17}$ or 4.12 m/s — 5

direction $= \tan^{-1}\left(\frac{1}{4}\right)$
or 14.04° with bank. — 5

(iv) time $= \frac{72}{4} = 18$ seconds — 10

(v) time $= \frac{72}{3} = 24$ seconds — 5

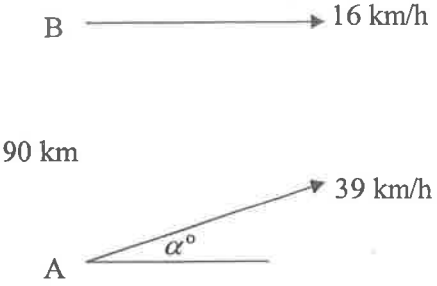
$\Rightarrow$ required time $= 6$ seconds — 5

50

2. Ship A is travelling east $\alpha°$ north with a constant speed of 39 km/h, where $\tan\alpha = \frac{5}{12}$.

Ship B is travelling due east with a constant speed of 16 km/h.

At 2 pm ship B is positioned 90 km due north of ship A.



206
(6)

(i) Express the velocity of ship A and the velocity of ship B in terms of $\vec{i}$ and $\vec{j}$.

(ii) Find the velocity of ship A relative to ship B in terms of $\vec{i}$ and $\vec{j}$.

(iii) Find the shortest distance between the ships.

(i) $\vec{V}_A = 39\cos\alpha\,\vec{i} + 39\sin\alpha\,\vec{j}$ — 5

$= 39\left(\frac{12}{13}\right)\vec{i} + 39\left(\frac{5}{13}\right)\vec{j}$

$= 36\vec{i} + 15\vec{j}$ — 5

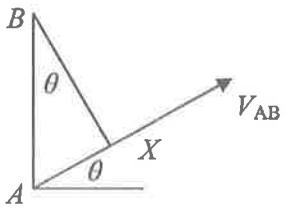
$\vec{V}_B = 16\vec{i} + 0\vec{j}$ — 10

(ii) $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$ — 5

$= (36\vec{i} + 15\vec{j}) - (16\vec{i})$ — 5,5

$= 20\vec{i} + 15\vec{j}$ — 5

(iii)



shortest distance $= |BX|$

$= 90\cos\theta$ — 5

$= 90\left(\frac{20}{25}\right)$

$= 72$ km — 5

50

2003
(O)

2. The velocity of ship A is $3\vec{i} - 4\vec{j}$ m/s and the velocity of ship B is $-2\vec{i} + 8\vec{j}$ m/s.

(i) Find the velocity of ship A relative to ship B in terms of $\vec{i}$ and $\vec{j}$.

(ii) Find the magnitude and direction of the velocity of ship A relative to ship B, giving the direction to the nearest degree.

At a certain instant, ship B is 26 km due east of ship A.

(iii) Show, on a diagram, the positions of ship A and ship B at this instant and show, also, the direction in which ship A is travelling relative to ship B.

(iv) Calculate the shortest distance between the ships, to the nearest km.

(i)
$$\begin{aligned} V_{AB} &= V_A - V_B \\ &= \left(3\vec{i} - 4\vec{j}\right) - \left(-2\vec{i} + 8\vec{j}\right) \\ &= 5\vec{i} - 12\vec{j} \end{aligned}$$

10

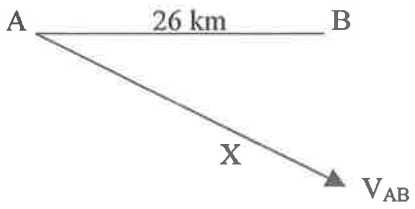
(ii)
$$\text{magnitude} = \sqrt{(5)^2 + (-12)^2}$$
$$= 13 \text{ m/s}$$

10

$$\text{direction} = \tan^{-1}\left(\frac{12}{5}\right)$$
$$= 67° \text{ south of east}$$

10

(iii)



10

(iv) Shortest distance $= |BX|$
$= 26 \sin 67°$
$= 24$ km

10

50

2. (a) Two boats, B and C, are each moving with constant velocity.
At a certain instant, boat B is 10 km due west of boat C.
The speed and direction of boat B relative to boat C is 2.5 m/s in the direction 60° south of east.

(i) Calculate the shortest distance between the boats, to the nearest metre.

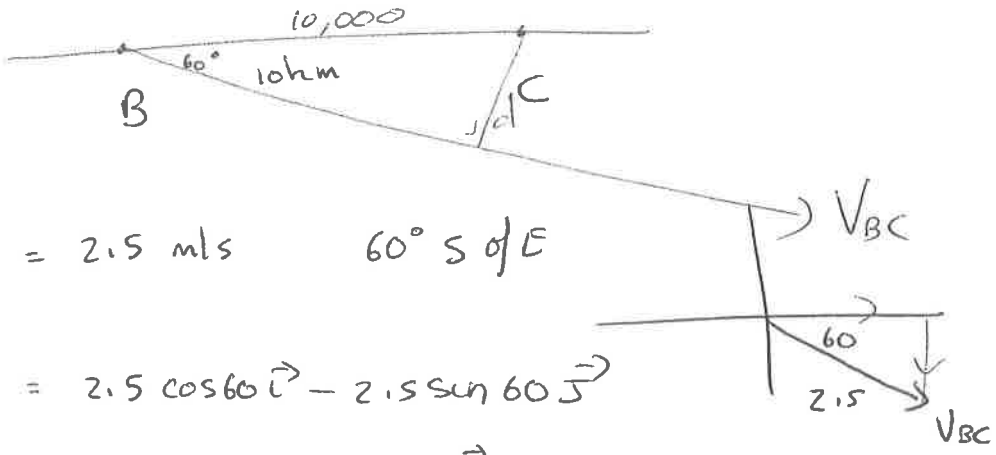
(ii) Calculate the length of time, to the nearest second, for which the boats are less than or equal to 9 km apart.

2002
(H)

(b) The velocity of ship P relative to a steady wind is 20 km/hr in the direction 80° north of east.
The velocity of ship Q relative to the same steady wind is 10 km/hr in the direction 20° south of west.

Calculate the magnitude and direction of the velocity of ship P relative to ship Q.
Give your answers to the nearest km and the nearest degree, respectively.

2002 a



$V_{BC} = 2.5$ m/s $\quad 60°$ S of E

$= 2.5\cos 60\,\vec{i} - 2.5\sin 60\,\vec{j}$

$= 1.25\vec{i} - \frac{5\sqrt{3}}{4}\vec{j}$

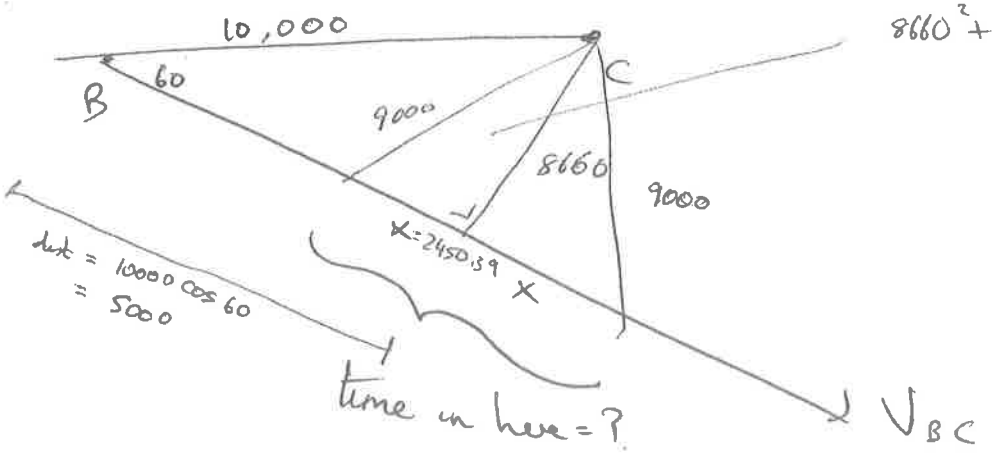
$\alpha = \tan^{-1}\dfrac{\frac{5\sqrt{3}}{4}}{1.25} = 60°$

$|V_{BC}| = \sqrt{1.25^2 + \left(\frac{5\sqrt{3}}{4}\right)^2}$

$= 5/2$

i) $d = 10{,}000\sin 60 = 8660$

ii)

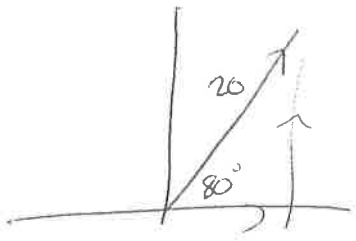


$8660^2 + x^2 = 9000^2$

$\text{time} = \dfrac{\text{dist}}{\text{speed}} = \dfrac{2(2450.39)}{5/2} = 1960.312$

2002 b

$V_{pw} = 20 \quad 80^\circ$ N of E

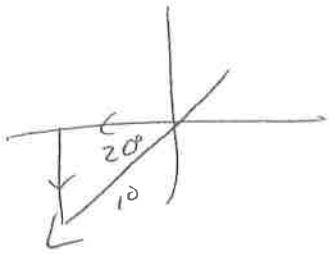


$V_{pw} = 20\cos 80\,\vec{i} + 20\sin 80\,\vec{j}$

$= 3.473\,\vec{i} + 19.70\,\vec{j}$

19.696

$V_{aw} = 10 \quad 20^\circ$ S of W



$V_{aw} = -10\cos 20\,\vec{i} - 10\sin 20\,\vec{j}$

$= -9.397\,\vec{i} - 3.420\,\vec{j}$

$V_{pw} = V_p - V_w$

$V_{aw} = V_a - V_w$

$V_{PA} = V_p - V_a$

$= (V_{pw} + V_w) - (V_{aw} + V_w)$

$= V_{pw} - V_{aw}$

$= 12.9\,\vec{i} + 23.1\,\vec{j}$

$|\ | = 26.457 = 26$ km/hr

$\tan\alpha = 1.7961 \Rightarrow 61^\circ$ N of E.