

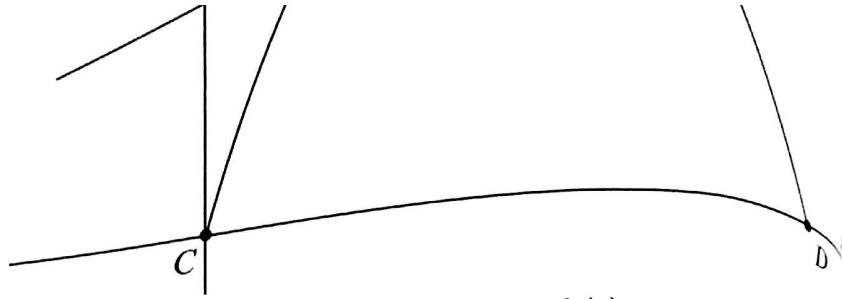
$$0 = (x-1)(x-3)$$

$$\therefore x = 1, 3$$

$$g(1) = 1 + 3 = 4$$

$$g(3) = 3 + 3 = 6$$

$$A(1, 4), B(3, 6)$$



Question 6 (b)

$$|A| = \int_1^3 (f(x) - g(x)) dx$$

$$= \int_1^3 (-x^2 + 4x - 3) dx$$

$$= \left[-\frac{1}{3}x^3 + 2x^2 - 3x\right]_1^3$$

$$= \left\{-\frac{1}{3}(3)^3 + 2(3)^2 - 3(3)\right\} - \left\{-\frac{1}{3}(1)^3 + 2(1)^2 - 3(1)\right\}$$

$$= -9 + 18 - 9 + \frac{1}{3} - 2 + 3$$

$$= \frac{4}{3}$$

Question 6 (c)

$C(0, 0), D(5, 0)$

$$\text{Average value} = \frac{1}{5-0} \int_0^5 f(x) dx$$

$$= \frac{1}{5} \int_0^5 (5x - x^2) dx$$

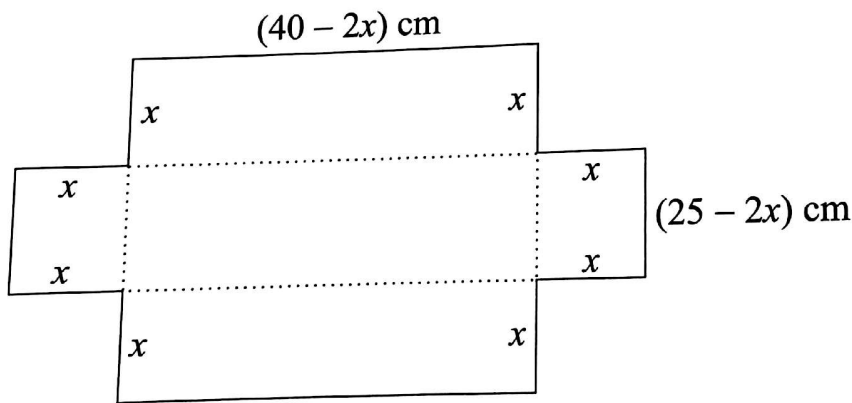
$$= \frac{1}{5} \left[\frac{5}{2}x^2 - \frac{1}{3}x^3\right]_0^5$$

$$= \frac{1}{5} \left\{\frac{5}{2}(5)^2 - \frac{1}{3}(5)^3\right\} - \{0\}$$

$$= \frac{1}{5} \left(\frac{125}{2} - \frac{125}{3}\right) = \frac{25}{6}$$

QUESTION 7 (50 MARKS)

Question 7 (a) (i)



$$l = 40 - 2x, b = 25 - 2x, h = x$$

Question 7 (b) (i)

$$S = 2(40 - 2x)x + 2(25 - 2x)x + (40 - 2x)(25 - 2x)$$

$$= 80x - 4x^2 + 50x - 4x^2 + 1000 - 80x - 50x + 4x^2$$

$$= 1000 - 4x^2$$

Question 7 (a) (ii)

$$V = (40 - 2x)(25 - 2x)x$$

$$= (1000 - 80x - 50x + 4x^2)x$$

$$= (1000 - 130x + 4x^2)x$$

$$= 1000x - 130x^2 + 4x^3$$

Question 7 (b) (ii)

$$600 = 1000 - 4x^2$$

$$4x^2 = 400$$

$$x^2 = 100$$

$$\therefore x = 10 \text{ cm}$$

Question 7 (c)

$$V = 4x^3 - 130x^2 + 1000x$$

$$4x^3 - 130x^2 + 1000x = 1512$$

$$2x^3 - 65x + 500x - 756 = 0$$

$$x = 1: 2(1)^3 - 65(1) + 500(1) - 756 = -319 \neq 0$$

$$x = 2: 2(2)^3 - 65(2) + 500(2) - 756 = 0$$

Question 7 (d)

$$V = 4x^3 - 130x^2 + 1000x$$

$$\frac{dV}{dx} = 12x^2 - 260x + 1000$$

Maximum Volume: $\frac{dV}{dx} = 0$

$$\frac{dV}{dx} = 0 \Rightarrow 12x^2 - 260x + 1000 = 0$$

$$3x^2 - 65x + 250 = 0$$

$$(3x - 50)(x - 5) = 0$$

$x = 5, \frac{50}{3}$ cm [Ignore the second solution as it will cause one of the sides to be negative.]

$$V_{\text{Max}} = 4(5)^3 - 130(5)^2 + 1000(5) = 500 - 3250 + 5000 = 2250 \text{ cm}^3$$

$$l = 40 - 2(5) = 30 \text{ cm}, b = 25 - 2(5) = 15 \text{ cm}, h = 5 \text{ cm}$$

Question 7 (e)

$$\frac{dV}{dt} = -(2t + 5)$$

$$V = -\int (2t + 5) dt = -t^2 - 5t + c$$

$$V = 2250 \text{ when } t = 0: 2250 = c$$

$$\therefore V = -t^2 - 5t + 2250$$

$$\begin{aligned} -t^2 - 5t + 2250 &= 0 \\ t^2 + 5t - 2250 &= 0 \\ (t - 45)(t + 50) &= 0 \\ \therefore t &= 45 \text{ s} \end{aligned}$$

Question 7 (f) (i)

$$\tan 45^\circ = \frac{r}{h} \Rightarrow 1 = \frac{r}{h}$$

$$\therefore r = h$$

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi r^3$$

Question 7 (f) (ii)

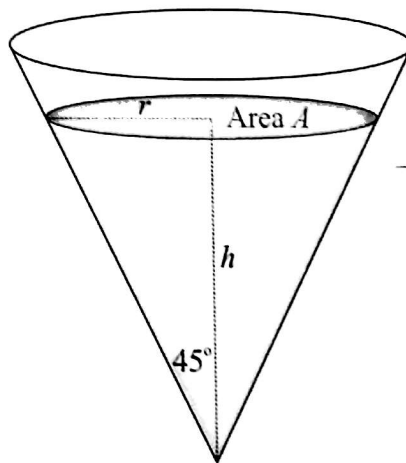
$$V = \frac{1}{3} \pi r^3$$

$$\frac{dV}{dt} = \pi r^2 \times \frac{dr}{dt} = 3 \Rightarrow \frac{dr}{dt} = \frac{3}{\pi r^2}$$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \times \frac{dr}{dt} = 2\pi r \times \frac{3}{\pi r^2} = \frac{6}{r}$$

$$\left(\frac{dA}{dt}\right)_{r=2} = \frac{6}{2} = 3 \text{ cm}^2/\text{s}$$



$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt} \quad A = \pi r^2 \quad \frac{dA}{dr} = 2\pi r$$

$$\frac{dv}{dt} = 3 \quad v = \frac{1}{3} \pi r^3 \quad \frac{dv}{dr} = \pi r^2$$

$$\frac{dv}{dt} = \frac{dv}{dr} \cdot \frac{dr}{dt} \quad 3 = \pi r^2 \cdot \frac{dr}{dt} \quad \frac{dr}{dt} = \frac{3}{\pi r^2}$$

$$\frac{dA}{dt} = 2\pi r \cdot \frac{3}{\pi r^2} = \frac{6}{r} = 3$$