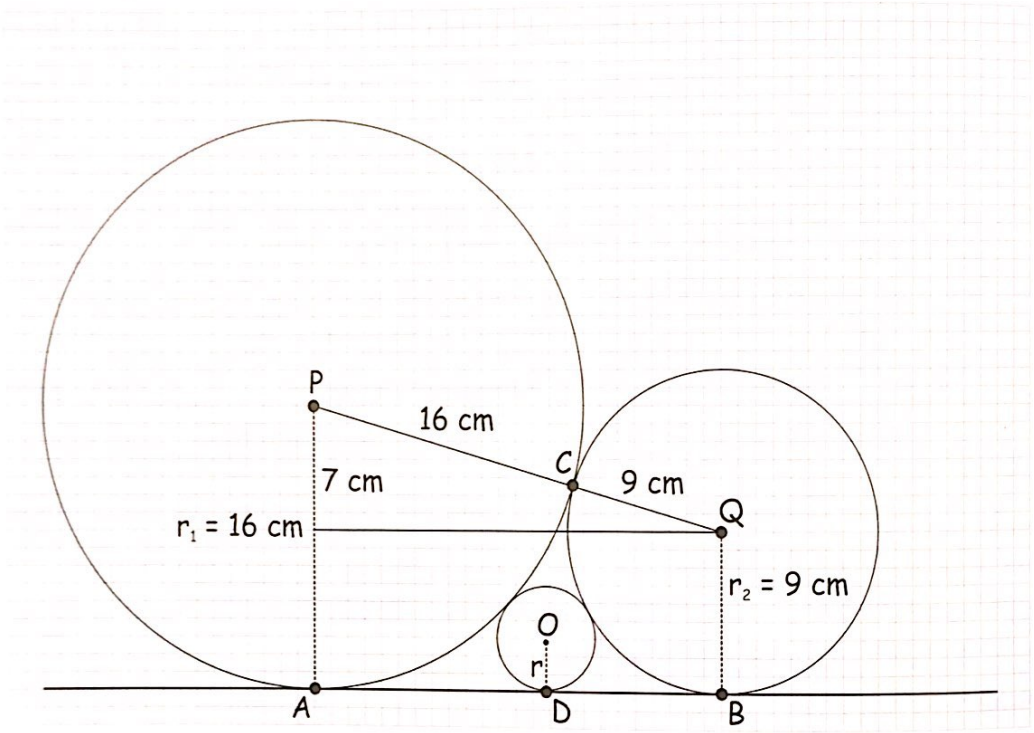
Question 8 (75 marks)

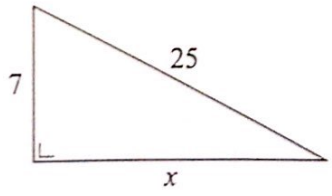


Question 8 (a) (i)

$r_1 = 16$ cm

$r_2 = 9$ cm

Question 8 (b) (i)



$x^2 + 7^2 = 25^2$

$x = \sqrt{25^2 - 7^2} = 24$ cm

Question 8 (a) (ii)

$|PQ| = |PC| + |CQ| = r_1 + r_2$

$= 16 \text{ cm} + 9 \text{ cm} = 25 \text{ cm}$

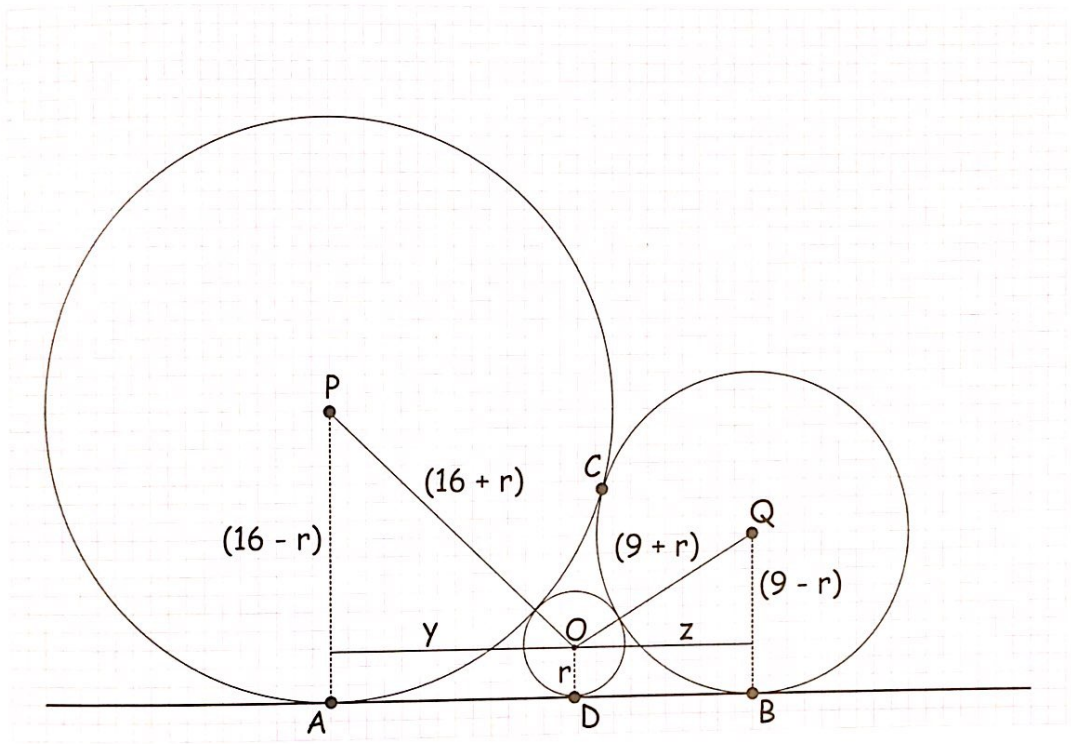
Question 8 (b) (ii)

$$\tan(|\angle APQ|) = \frac{24}{7}$$

$$\therefore |\angle APQ| = \tan^{-1}\left(\frac{24}{7}\right)$$

$$= 73.74^\circ = 1.287 \text{ rads}$$

$$A = \tfrac{1}{2} r^2\theta = \tfrac{1}{2}(16)^2(1.287) = 164.7 \text{ cm}^2$$



Question 8 (c) (i)

$y^2 + (16-r)^2 = (16+r)^2$

$y^2 = (16+r)^2 - (16-r)^2$

$y^2 = (16+r+16-r)(16+r-16+r)$

$y^2 = (32)(2r) = 64r$

$y = 8\sqrt{r}$

Question 8 (c) (ii)

$z^2 + (9-r)^2 = (9+r)^2$

$z^2 = (9+r)^2 - (9-r)^2$

$z^2 = (9+r+9-r)(9+r-9+r)$

$z^2 = (18)(2r) = 36r$

$z = 6\sqrt{r}$

Question 8 (d)

$y + z = 24$

$8\sqrt{r} + 6\sqrt{r} = 24$

$14\sqrt{r} = 24$

$\sqrt{r} = \dfrac{24}{14} = \dfrac{12}{7}$

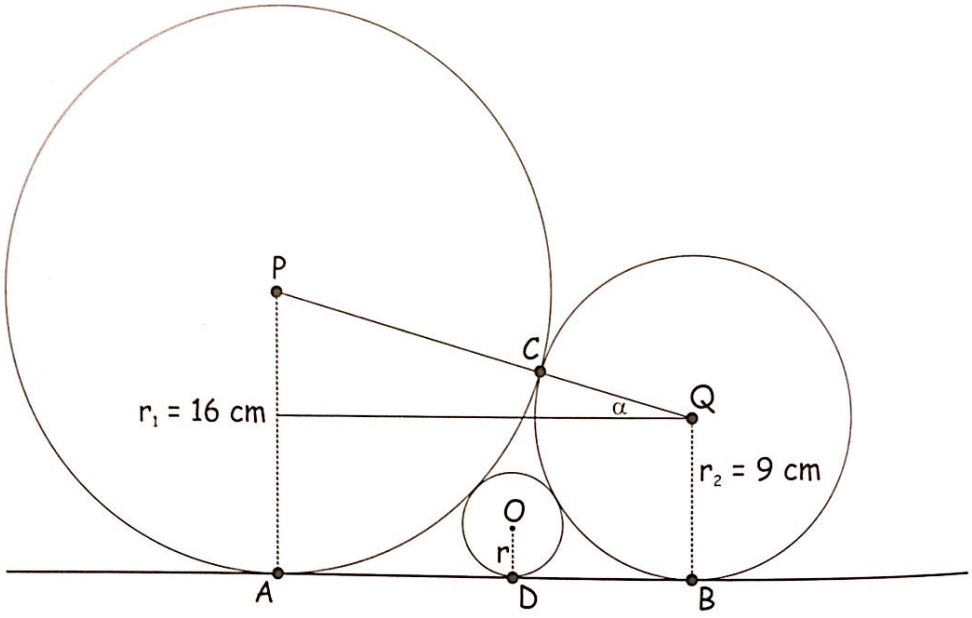
$\therefore r = \dfrac{144}{49}$

Question 8 (e)

Area of trapezium $ABQP$: $A = \frac{1}{2}(16+9)(24) = 25(12) = 300 \text{ cm}^2$

Question 8 (f)

Area of space between wheels
= Area of trapezium – Area of sector APC – Area of sector CQB – Area of smallest wheel



Area of sector CQB:

$\alpha = 90^\circ - 73.74^\circ = 16.26^\circ$

$|\angle CQB| = 16.26^\circ + 90^\circ = 106.26^\circ = 1.855 \text{ rads}$

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(9)^2(1.855) = 75.13 \text{ cm}^2$

Area of space between wheels $= 300 - 164.7 - 75.13 - \pi\left(\frac{144}{49}\right)^2 \approx 33 \text{ cm}^2$

% of air space to area of three wheels $= \dfrac{33}{\pi\left[16^2 + 9^2 + \left(\frac{144}{49}\right)^2\right]} \times 100\% \approx 3\%$