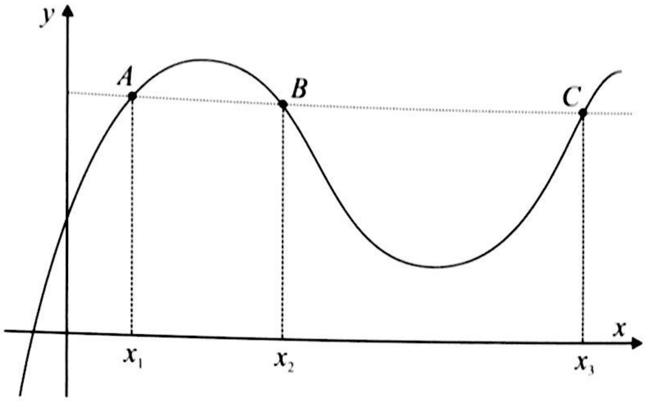
Question 5 (25 marks)



Question 5 (a)

It is not injective because there is more than one x value mapping on to the same y value.

It is surjective because every y value has a corresonding x value.

Question 5 (b)

$f(x) = y = \frac{\ln x}{x}$

$\frac{dy}{dx} = \frac{x(\frac{1}{x}) - (\ln x)(1)}{x^2} = \frac{1-\ln x}{x^2}$

$\frac{dy}{dx} = 0 \Rightarrow \frac{1-\ln x}{x^2} = 0$

$1 - \ln x = 0$

$1 = \ln x \Rightarrow x = e$

$f(e) = \frac{\ln e}{e} = \frac{1}{e}$

TP: $\left(e, \frac{1}{e}\right)$

Question 5 (c)

$\frac{dy}{dx} = \frac{1-\ln x}{x^2}$

$\frac{d^2y}{dx^2} = \frac{x^2(-\frac{1}{x}) - (1-\ln x)(2x)}{(x^2)^2}$

$= \frac{-x - 2x + 2x\ln x}{x^4}$

$= \frac{-3x + 2x\ln x}{x^4}$

$\left(\frac{d^2y}{dx^2}\right)_{x=e} = \frac{-3e + 2e\ln e}{e^4} = \frac{-3e+2e}{e^4} = \frac{-e}{e^4} = -\frac{1}{e^3} < 0$

$\therefore \left(e, \frac{1}{e}\right)$ is a local maximum

Question 5 (d)

$a^b = b^a$

$\ln a^b = \ln b^a$

$b \ln a = a \ln b$

$\therefore \frac{\ln a}{a} = \frac{\ln b}{b}$

Question 5 (e) (i)

$2^4 = 4^2 \Rightarrow \frac{\ln 2}{2} = \frac{\ln 4}{4}$

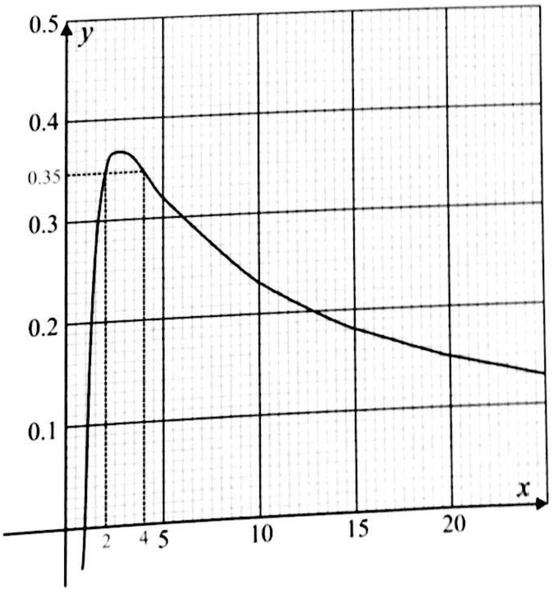
$f(x) = \frac{\ln x}{x}$

$f(2) = \frac{\ln 2}{2}$

$f(4) = \frac{\ln 4}{4}$

$\therefore f(2) = f(4) = 0.35$

Calculator: $\frac{\ln 2}{2} = \frac{\ln 4}{4} = 0.3466$



Question 5 (e) (ii)

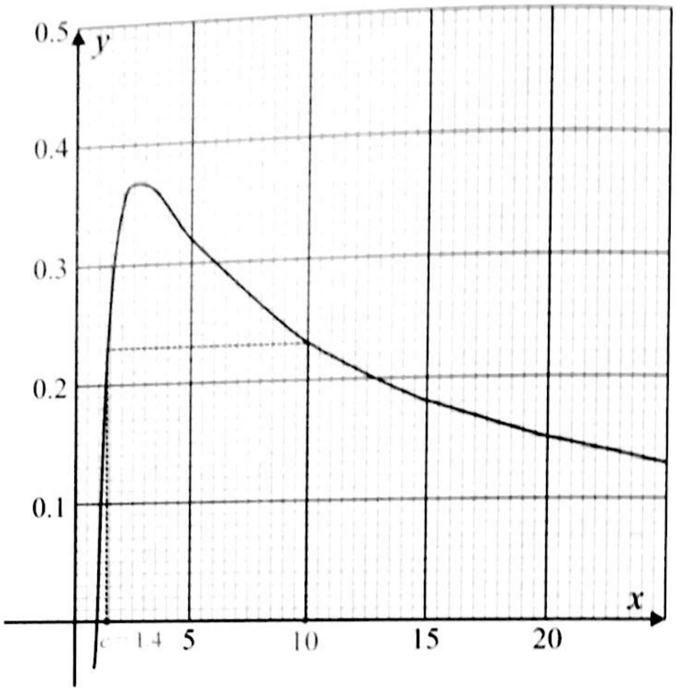
$10^c = c^{10} \Rightarrow \frac{\ln 10}{10} = \frac{\ln c}{c}$

$f(x) = \frac{\ln x}{x}$

$f(10) = \frac{\ln 10}{10}$

$f(c) = \frac{\ln c}{c}$

$\therefore f(10) = f(c) \Rightarrow c \approx 1.4$



Question 5 (e) (iii)

The maximum value of $f(x)$ as calculated in **part (b)** is e.

$\therefore e \approx 2.7$