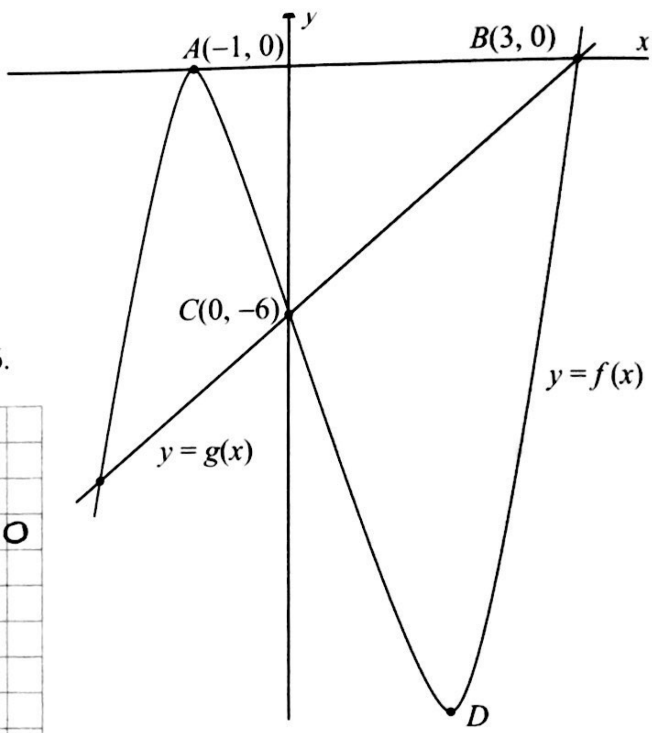
Question 6 **(25 marks)**

The function $f(x) = ax^3 + bx^2 + cx + d$ is graphed as shown.



(a) A and D are turning points of $f(x)$.

B is the intercept of $f(x)$ on the x-axis.

C is the intercept of $f(x)$ on the y-axis.

Show that $a = 2$, $b = -2$, $c = -10$ and $d = -6$.

$(-1, 0)$ on $f(x)$

$\Rightarrow a(-1)^3 + b(-1)^2 + c(-1) + d = 0$

$\Rightarrow \boxed{-a + b - c + d = 0}$ ①

$(0, -6)$ on $f(x)$

$\Rightarrow a(0)^3 + b(0)^2 + c(0) + d = -6$

$\Rightarrow \boxed{d = -6}$ ② Sub in ① $\Rightarrow \boxed{-a + b - c = 6}$ ③

$(3, 0)$ on $f(x)$

$\Rightarrow a(3)^3 + b(3)^2 + c(3) + d = 0$

$\Rightarrow \boxed{27a + 9b + 3c + d = 0}$ ④ $\Rightarrow \boxed{27a + 9b + 3c = 6}$ ⑥

$f'(x) = 3ax^2 + 2bx + c$

$f'(-1) = 3a(-1)^2 + 2b(-1) + c = 0$ (turning point)

$\Rightarrow \boxed{3a - 2b + c = 0}$ ⑤

$2\times$③ $-2a + 2b - 2c = 12$

$\boxed{a - c = 12}$ ⑦

⑥ $\Rightarrow 27a + 9b + 3c = 6$

$-9\times$③ $\Rightarrow 9a - 9b + 9c = -54$

$36a + 12c = -48$

$\therefore \boxed{3a + c = -4}$ ⑧

⑧ $\Rightarrow$ $3a + c = -4$

⑦ $\Rightarrow$ $a - c = 12$

$4a = 8$

$\boxed{a = 2}$

Sub in ⑦

$2 - c = 12$

$\boxed{-10 = c}$

Sub in ⑤

$3(2) - 2b + (-10) = 0$

$\therefore -4 = 2b$

$\boxed{-2 = b}$

(b) Find the x-coordinate of D.

$$f'(x) = 3(2)x^2 + 2(-2)x + (-10) = 0$$

$$\therefore\ 6x^2 - 4x - 10 = 0$$

$$\therefore\ 3x^2 - 2x - 5 = 0$$

$$(3x-5)(x+1) = 0$$

$$\boxed{x = \frac{5}{3}} \quad \text{or} \quad x = -1$$

This is point A

(c) $g(x)$ is a straight line going through BC. Find its equation.

$$\text{Slope } BC = \frac{0+6}{3-0} = 2$$

$$\therefore\ y - 0 = 2(x-3)$$

$$y = 2x - 6$$

$$\boxed{0 = 2x - y - 6}$$

(d) Find the area of the shaded region.

$$\text{Area} = \int_0^3 (y_{high} - y_{low})\,dx$$

$$= \int_0^3 (2x-6) - (2x^3 - 2x^2 - 10x - 6)\,dx$$

$$= \int_0^3 (-2x^3 + 2x^2 + 12x)\,dx$$

$$= \left[-\frac{\cancel{2}x^4}{\cancel{4}_2} + \frac{2x^3}{3} + \frac{\cancel{12}^{6}x^2}{\cancel{2}}\right]_0^3$$

$$= -\frac{3^4}{2} + \frac{2(3)^3}{3} + 6(3)^2 - 0$$

$$= \boxed{\frac{63}{2}}$$

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Sample 3 Paper 1