

QUESTION 9 (50 MARKS)

Question 9 (a) (i)

$$V = 16\pi = \pi r^2 h \Rightarrow h = \frac{16}{r^2}$$

Question 9 (a) (ii)

$$\begin{aligned} A &= 2\pi r^2 + 2\pi r h \\ &= 2\pi r^2 + 2\pi r \left(\frac{16}{r^2}\right) \\ &= 2\pi r^2 + \frac{32\pi}{r} \end{aligned}$$

Question 9 (a) (iii)

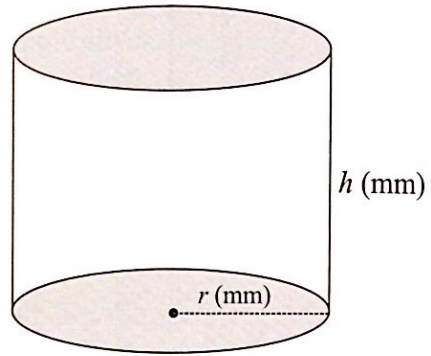
$$\begin{aligned} A &= 2\pi r^2 + 32\pi r^{-1} \\ \frac{dA}{dr} &= 0 \Rightarrow 4\pi r - 32\pi r^{-2} = 0 \\ 4\pi r - \frac{32\pi}{r^2} &= 0 \Rightarrow r^3 = 8 \\ \therefore r &= 2 \text{ mm} \end{aligned}$$

Question 9 (b) (i)

$$\begin{aligned} \frac{dm}{dt} &= -e^{-\frac{1}{10}t} \\ \int dm &= -\int e^{-\frac{1}{10}t} dt \\ m &= 10e^{-\frac{1}{10}t} + c \\ t = 0, m = 10 : 10 &= 10e^0 + c \Rightarrow c = 0 \\ \therefore m &= 10e^{-\frac{1}{10}t} \text{ mg} \end{aligned}$$

Question 9 (b) (iv)

$$\begin{aligned} 2 &= 10e^{-\frac{1}{10}T} + 10e^{-\frac{1}{10}(T+5)} \\ 0.2 &= e^{-\frac{1}{10}T} (1 + e^{-\frac{1}{2}}) \\ e^{-\frac{1}{10}T} &= \frac{0.2}{1 + \frac{1}{\sqrt{e}}} = 0.12449 \\ -\frac{1}{10}T &= \ln(0.12449) \\ \therefore T &= -10\ln(0.12449) = 20.835 \text{ hours} \end{aligned}$$



Question 9 (a) (iv)

$$\begin{aligned} A &= 2\pi r^2 + \frac{32\pi}{r} \\ A_{\text{Min.}} &= 2\pi(2)^2 + \frac{32\pi}{2} = 8\pi + 16\pi = 24\pi \text{ mm}^2 \end{aligned}$$

Question 9 (b) (ii)

$$t = 5 : m = 10e^{-\frac{1}{10}(5)} = 10e^{-0.5} = 6.065 \text{ mg}$$

Question 9 (b) (iii)

$$m = 10e^{-\frac{1}{10}} + 10e^{-\frac{6}{10}} = 14.536 \text{ mg}$$