QUESTION 1 (25 MARKS)

Question 1 (a)

$s : (x-2)^2 + (y+3)^2 = 25$

Centre $O(2, -3)$, $r = \sqrt{25} = 5$

Centre (h, k), Radius r
$(x-h)^2 + (y-k)^2 = r^2$

Substitute A into s to see if it satisfies the equation of the circle.
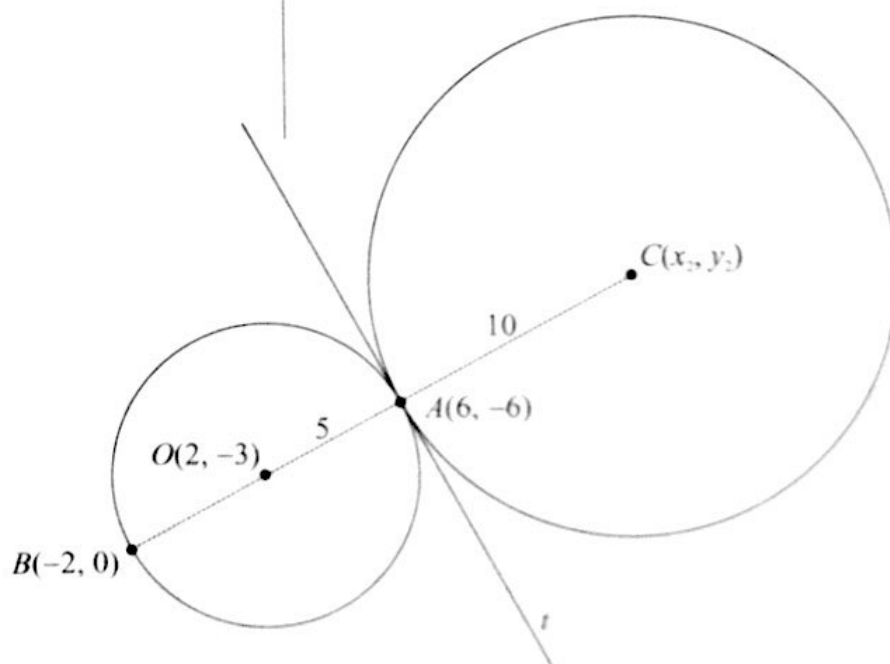
$A(6, -6) \in s$?

$(6-2)^2 + (-6+3)^2$

$= 16 + 9$

$= 25$

Pass A through O by a central symmetry to find B.

$A(6, -6) \to O(2, -3) \to B(-2, 0)$

Question 1 (b)

Slope of OA: $m_1 = \dfrac{-6-(-3)}{6-2} = \dfrac{-3}{4} = -\dfrac{3}{4}$

Slope of t: $m_2 = \dfrac{4}{3}$ [Perpendicular slope]

Equation of t

$m = \dfrac{4}{3}$, $(x_1, y_1) = A(6, -6)$

$y + 6 = \dfrac{4}{3}(x - 6)$

$3y + 18 = 4x - 24$

$t: 4x - 3y - 42 = 0$

Question 1 (c)

OC is divided in the ratio $a:b = 1:2$

$O(2, -3) = (x_1, y_1)$, $C(x_2, y_2)$, $A(6, -6)$

Ratio: $a:b = 1:2$

Point dividing $[PQ]$ in the ratio $a:b$	$\left(\dfrac{bx_1 + ax_2}{b+a}, \dfrac{by_1 + ay_2}{b+a}\right)$

$\therefore \dfrac{2(2) + 1x_2}{2+1} = 6 \Rightarrow 4 + x_2 = 18 \Rightarrow x_2 = 14$

$\therefore \dfrac{2(-3) + 1y_2}{2+1} = -6 \Rightarrow -6 + y_2 = -18 \Rightarrow y_2 = -12$

$\therefore C(14, -12)$

Equation of second circle: Centre $C(14, -12)$, $r = 10$

$(x-14)^2 + (y+12)^2 = 100$

Centre (h, k), Radius r
$(x-h)^2 + (y-k)^2 = r^2$

Mathematics Leaving Certificate