Trig

Exercise A:

1 Convert to degrees and minutes, correct to the nearest minute:

a) 15.75°   b) 35.5°   c) 86.25°   d) 45.4°   e) 33.33°

2. Convert to decimal degrees, correct to one decimal place:

a) 17°42′   b) 49°56′   c) 22°22′   d) 82°16′   e) 3°36′

f) 85°24′

3 Simplify, give answer in degrees and minutes:

a) 24°56′ + 39°42′
b) 52°11′ + 19°53′
c) 62°44′ − 7°52′
d) 13°13′ + 18°37′
e) 81°16′ − 3°3′
f) 45°19′ + 24°31′
g) 22°14′ − 14°31′
h) 63°19′ + 22°11′

4. Evaluate, to 4 decimal places:

a) sin 32°14′
b) tan 46°15′
c) cos 86°28′
d) tan 16°16′
e) cos 26°52′
f) sin 34°53′
g) tan 44°36′
h) sin 48°12′
i) cos 47.24°

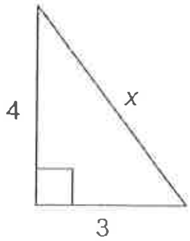
5 Two angles in a △ measure 27°52′ and 54°43′. Find the third angle.

6. Two angles in an isosceles △ both measure 46°32′. Find the third angle

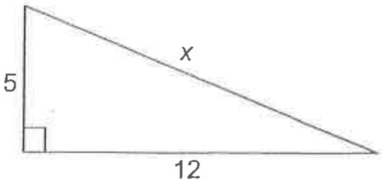
Exercise 11.1

1. Find the value of x in each case:

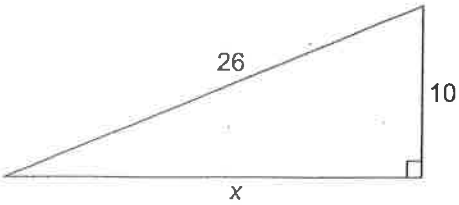
(i)



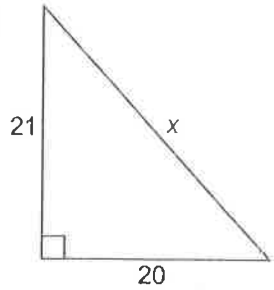
(ii)



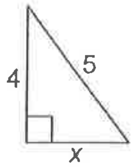
(iii)



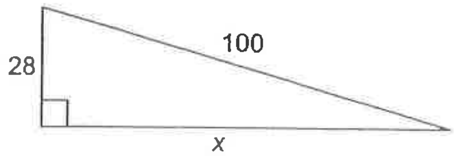
(iv)



(v)

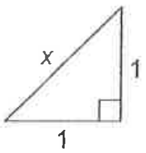


(vi)

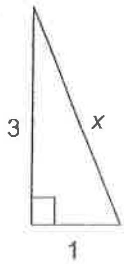


2. Find the value of x in each case (leave your answers in surd form):

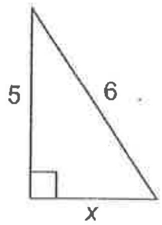
(i)



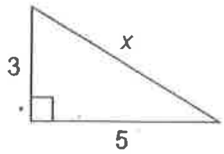
(ii)



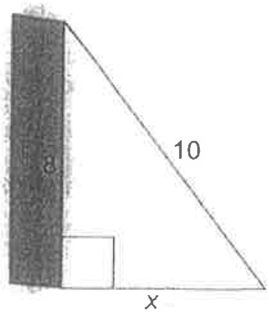
(iii)



(iv)

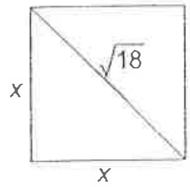


3. A ladder is 10 m long and rests against a vertical wall. The top of the ladder reaches a point on the wall which is 8 m above the ground. Find the distance from the wall to the foot of the ladder.

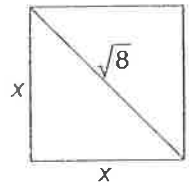


4. Use the theorem of Pythagoras to find the value of x:

(i)

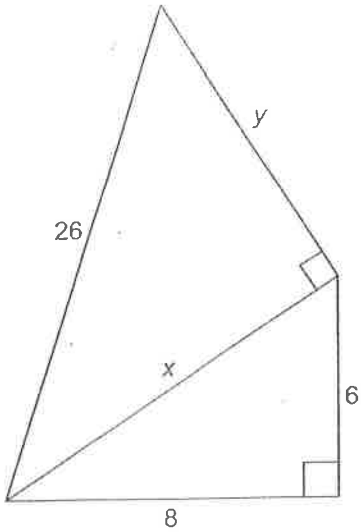


(ii)

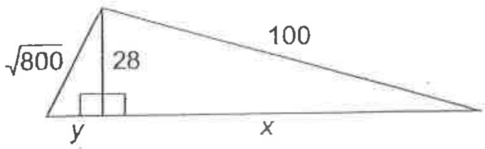


5. Find the value of x and y in each case:

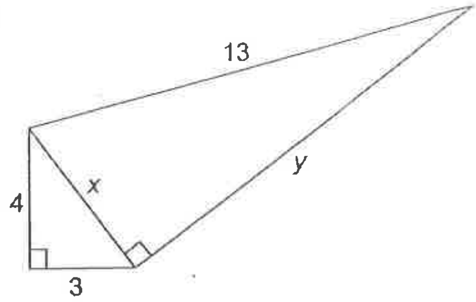
(i)



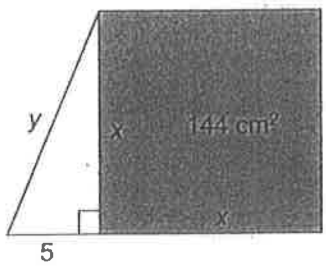
(ii)



(iii)



(iv)



6. How long is the diagonal of a square with a side of length 4 m? Correct your answer to two decimal places.

7. A rectangle has length 80 cm. Its perimeter has length 280 cm. Find the length of a diagonal of the rectangle.

8. The sides of a triangle are of lengths 85, 77 and 36. By applying the theorem of Pythagoras, investigate if the triangle is right-angled.

9. The sides of a triangle are of lengths 3,530, 3,526 and 168. By applying the theorem of Pythagoras, investigate if the triangle is right-angled.

10. Health and Safety regulations state that a 5 m ladder should be placed 1.3 m from the foot of the wall. How far up the wall does the ladder reach? Give your answer to three significant figures.

(b) To find cos 45°, key the following into your calculator:

$= 0.707106781$

$= 0.7071$ correct to four decimal places

(c) To find tan 22°, key the following into your calculator:

$= 0.404026225$

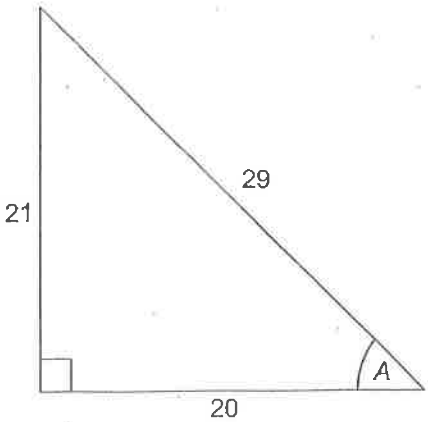
$= 0.4040$ correct to four decimal places

ACTIVITY 11.3

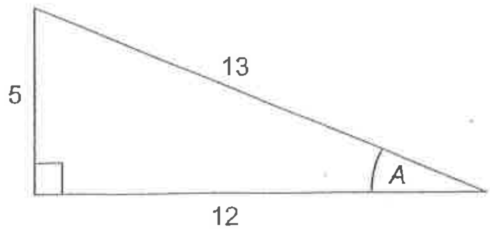
Exercise 11.2

1. For each of the following triangles, write down the values of sin A, cos A, and tan A.

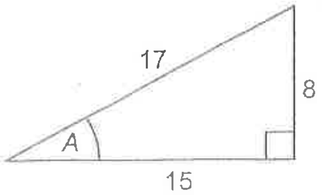
(i)



(ii)

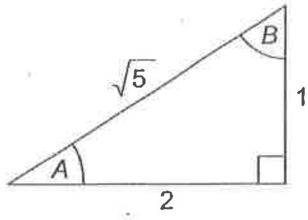


(iii)

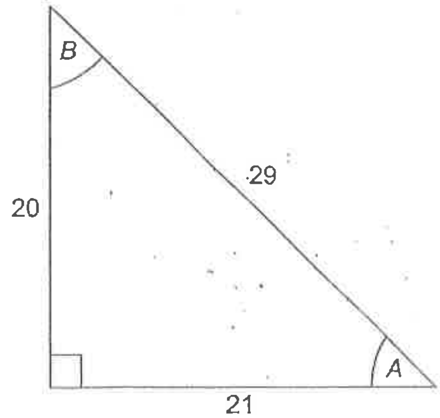


2. For each of the following triangles, write down the values of sin A, cos A, tan A, sin B, cos B and tan B.

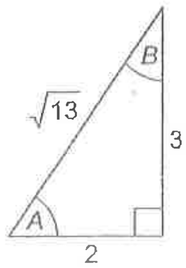
(i)



(ii)

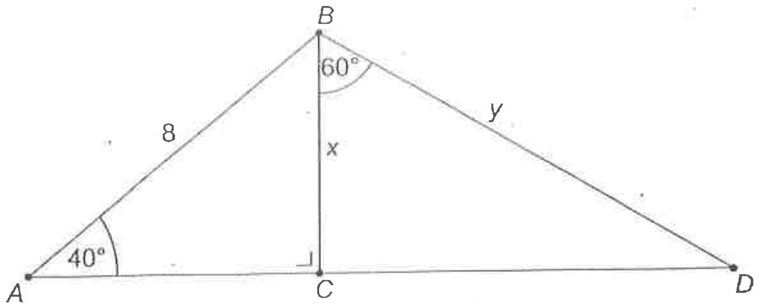


(iii)



Worked Example 11.6

Find (i) $|BC|$ and (ii) $|BD|$. Give your answers to two decimal places.



Solution

(i) Let $|BC| = x$

$$\sin 40° = \frac{x}{8}$$

$x = 8 \sin 40°$ (cross-multiplying)

$x = 5.14$ (calculator)

(ii) Let $|BD| = y$

$$\cos 60° = \frac{5.14}{y}$$

$y \cos 60° = 5.14$ (cross-multiplying)

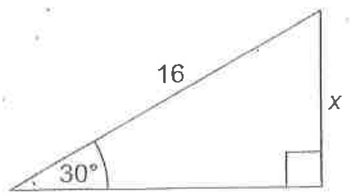
$$y = \frac{5.14}{\cos 60°}$$

$y = 10.28$ (calculator)

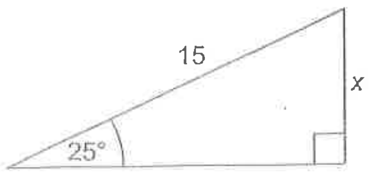
Exercise 11.3

1. Find the value of x in the following triangles (where necessary, round to two decimal places):

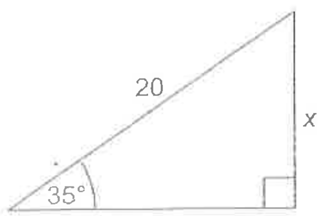
(i)



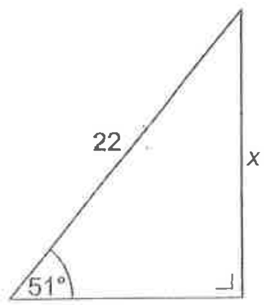
(ii)



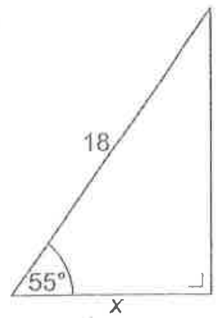
(iii)



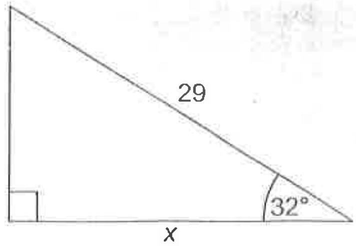
(iv)



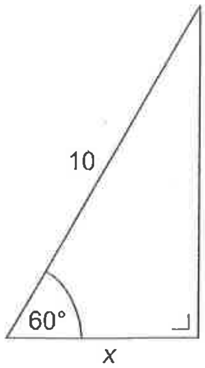
(v)



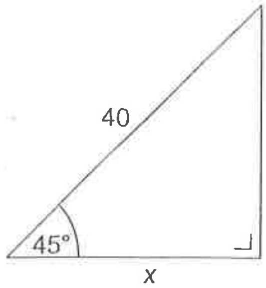
(vi)



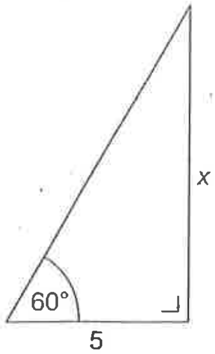
(vii)



(viii)

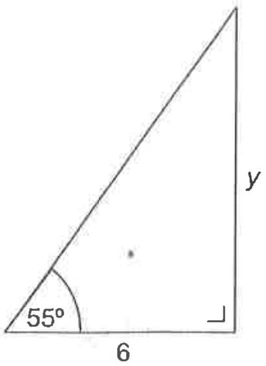


(ix)

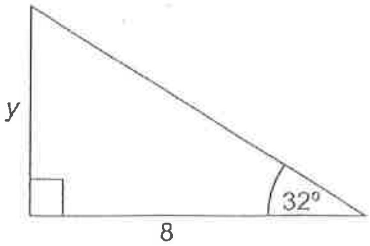


2. Find the value of y in the following triangles. Give your answers to two decimal places.

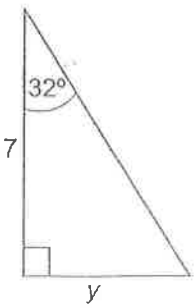
(i)



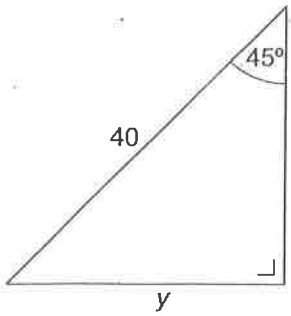
(ii)



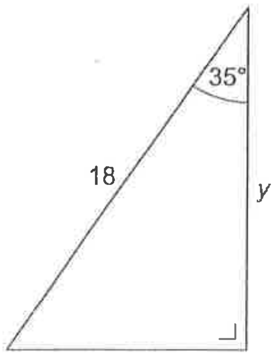
(iii)



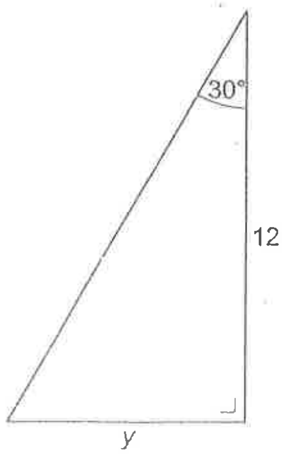
(iv)



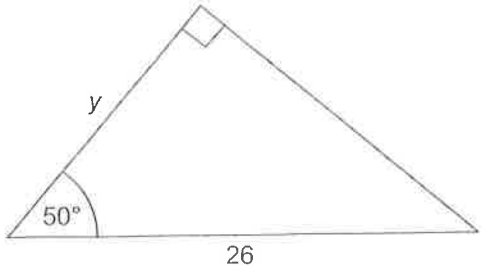
(v)



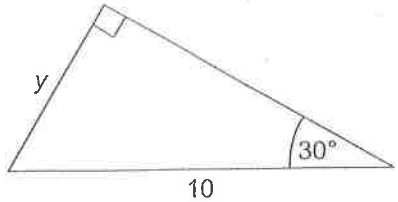
(vi)



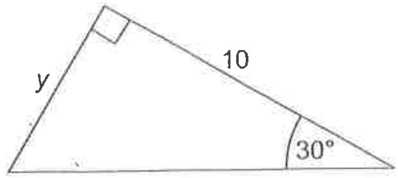
(vii)



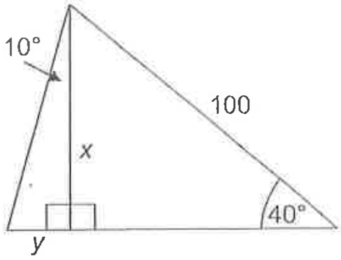
(viii)



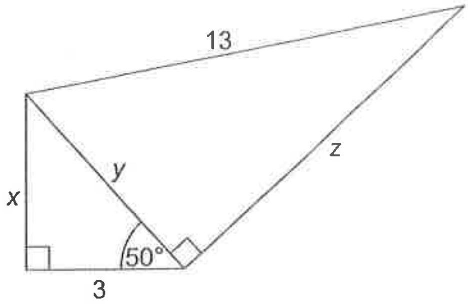
(ix)



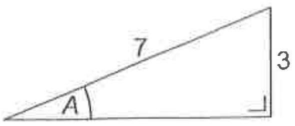
3. Solve for x and y to two decimal places:



4. Solve for x, y and z to two significant figures:



11.4 FINDING ANGLES



How can we find the measure of the angle A?

From the triangle we can see that $\sin A = \frac{3}{7} = 0.42857$.

We can now use the calculator to find A.

Key in the following:

2nd F | sin | 0.42857 | =

This gives an answer 25.37684293, which is 25° to the nearest degree.

Worked Example 11.7

If $\tan A = 0.7096$, use your calculator to find the measure of the angle A to the nearest degree.

Solution

Key in the following:

2nd F | tan | 0.7096 | =

This gives an answer 35.35951174, which is 35° to the nearest degree.

Exercise 11.4

1. Use your calculator to find the measure of the angle X. Give your answer to the nearest degree.

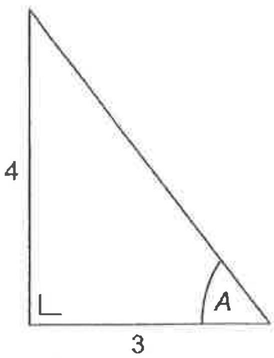
(i) $\sin X = 0.3452$
(ii) $\cos X = 0.7659$
(iii) $\tan X = 0.5467$
(iv) $\sin X = 0.4521$
(v) $\cos X = 0.6593$
(vi) $\tan X = 0.4678$
(vii) $\sin X = 0.7654$
(viii) $\cos X = 0.8345$
(ix) $\tan X = 0.9128$
(x) $\sin X = 0.1538$
(xi) $\cos X = 0.3472$
(xii) $\tan X = 0.4523$

2. Use your calculator to find the measure of the angle X. Give your answer to the nearest degree.

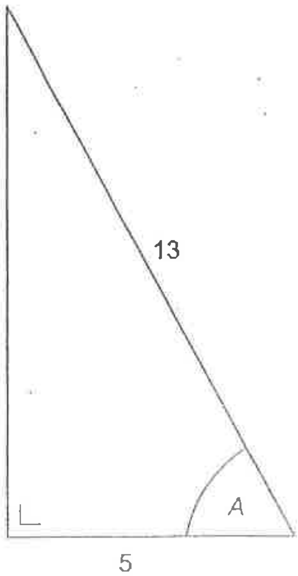
(i) $\sin X = 0.2543$
(ii) $\cos X = 0.9567$
(iii) $\tan X = 0.7645$
(iv) $\sin X = 0.1254$
(v) $\cos X = 0.3956$
(vi) $\tan X = 0.8764$
(vii) $\sin X = 0.1236$
(viii) $\cos X = 0.4376$
(ix) $\tan X = 0.5275$
(x) $\sin X = 0.6431$

3. Calculate to the nearest degree the value of the angle A:

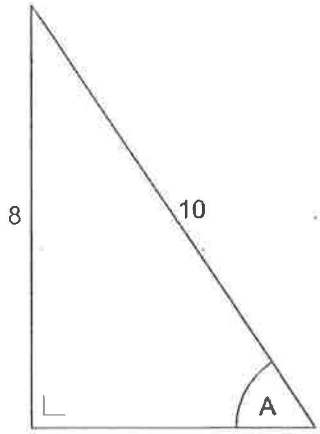
(i)



(ii)

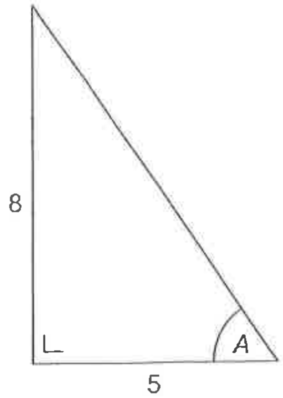


(iii)

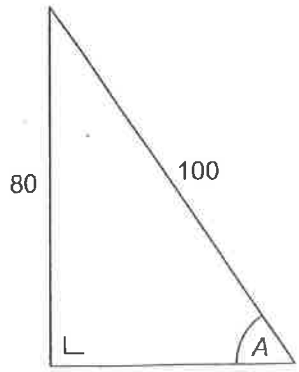


4. Calculate to the nearest degree the value of the angle A:

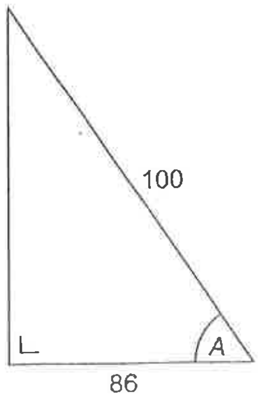
(i)



(ii)

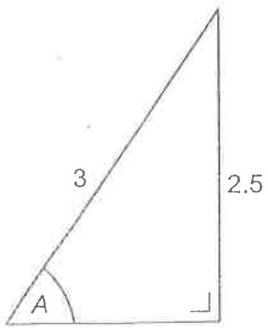


(iii)

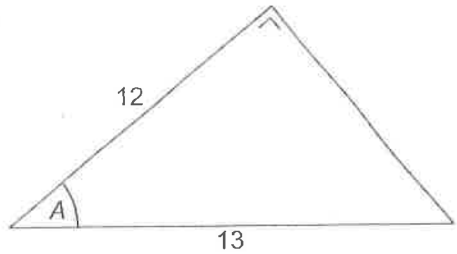


5. Calculate to the nearest degree, the value of the angle A:

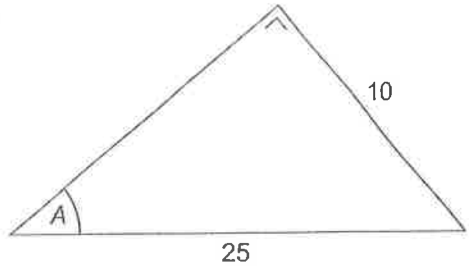
(i)



(ii)

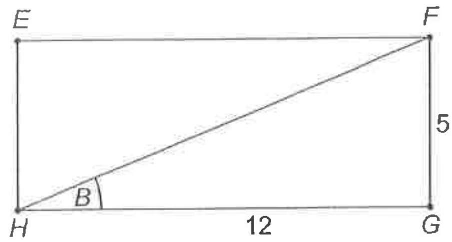


(iii)

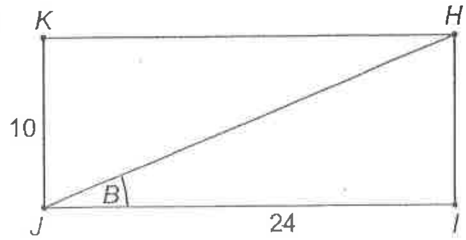


6. In the following parallelograms, calculate to the nearest degree the value of the angle B:

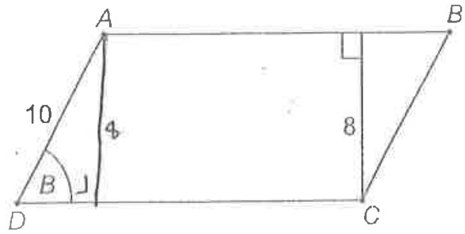
(i)



(ii)

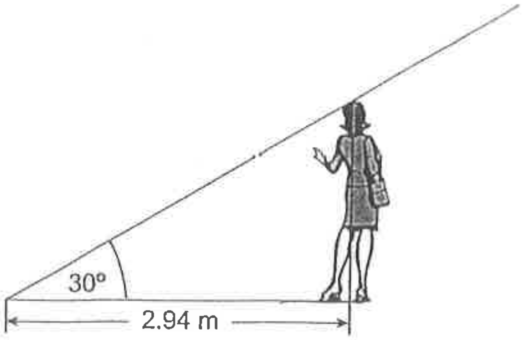


(iii)



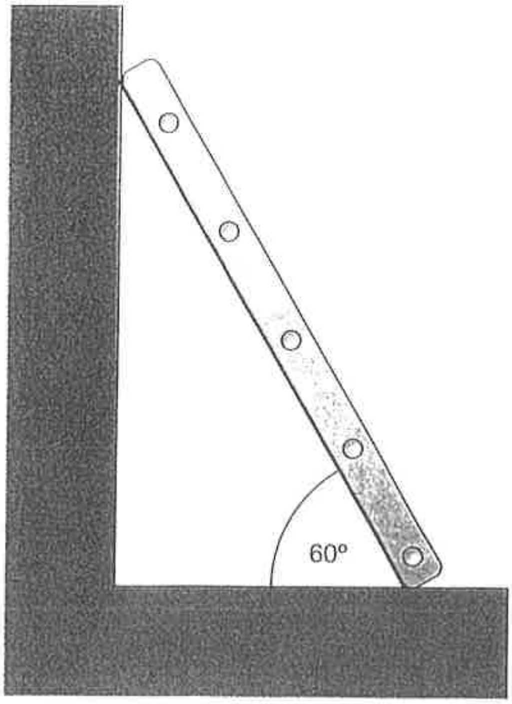
Exercise 11.5

1. A girl stands on level ground when the sun's elevation is 30°. The girl casts a shadow of length 2.94 m. Find the girl's height to the nearest centimetre.

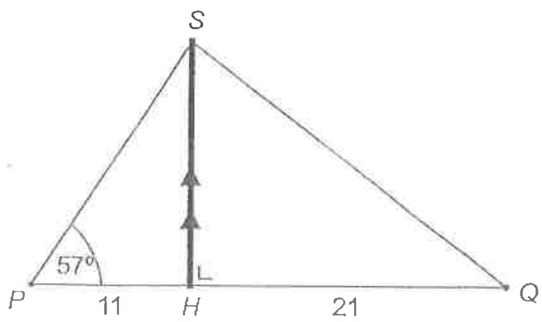


2. The Statue of Liberty pictured below is one of New York's tallest structures. Using the information given, calculate the height of the structure.

3. A ladder rests against a vertical wall. The ladder makes an angle of 60° with the horizontal and reaches a point on the wall 4 m above the ground. Find to the nearest centimetre the distance from the foot of the ladder to the wall.



4. P and Q are two islands 32 km apart in an east–west direction. H is a small island between P and Q, 11 km from P and 21 km from Q.



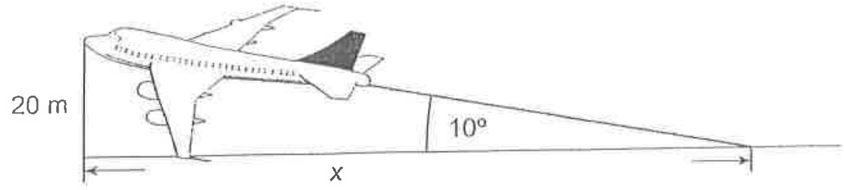
A ship sails north from H for two hours until it reaches a point S, as shown. Find (correct to one decimal place):

(i) The distance travelled by the ship

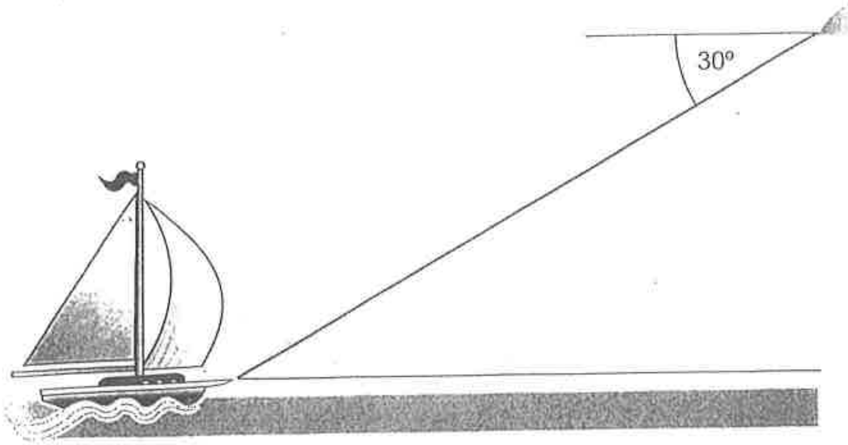
(ii) The speed of the ship

(iii) The distance $|SQ|$

5. A plane takes off from a level runway at an angle of 10° (see diagram). Calculate the value of x when the nose of the plane is 20 m above the runway.



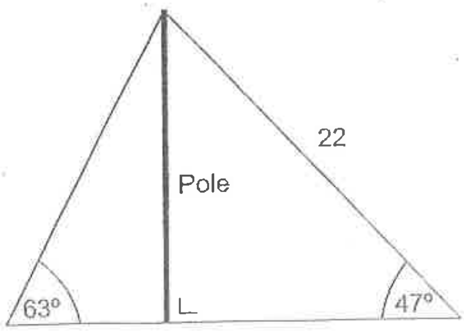
6. John is standing on a cliff top and observes a boat drifting towards the base of the cliff. He decides to call the emergency services and give them the position of the boat. He measures the angle of depression of the boat from the cliff top to be 30°, and he knows the cliff top is 200 m above sea level. How far is the boat from the base of the cliff?



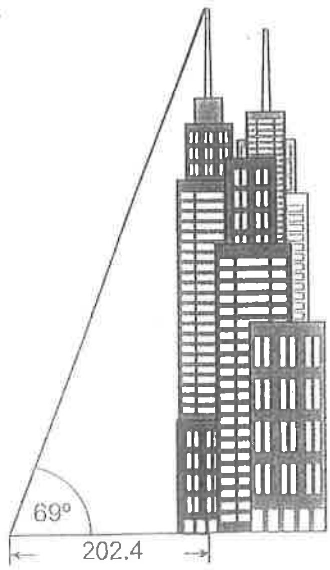
7. A vertical pole is tied to the horizontal ground by means of two wires. The longer wire is 22 m long and makes an angle of 47° with the ground. The shorter wire makes an angle of 63° with the ground. Find to the nearest metre:

(i) The height of the pole

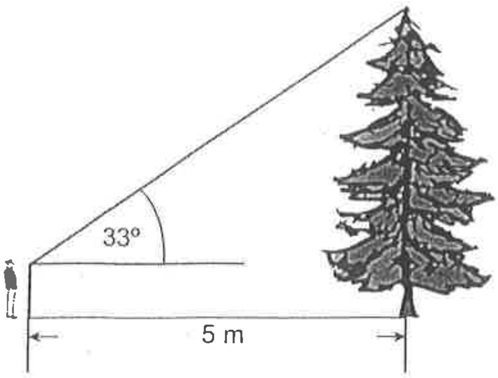
(ii) The length of the shorter wire



8. The Sears Tower in Chicago is one of the world's tallest structures. A tourist wishing to calculate the height of the tower makes the measurements shown in the diagram. Using these measurements, calculate the height of the tower. Using the Internet or some other source, find out the height of the tower.

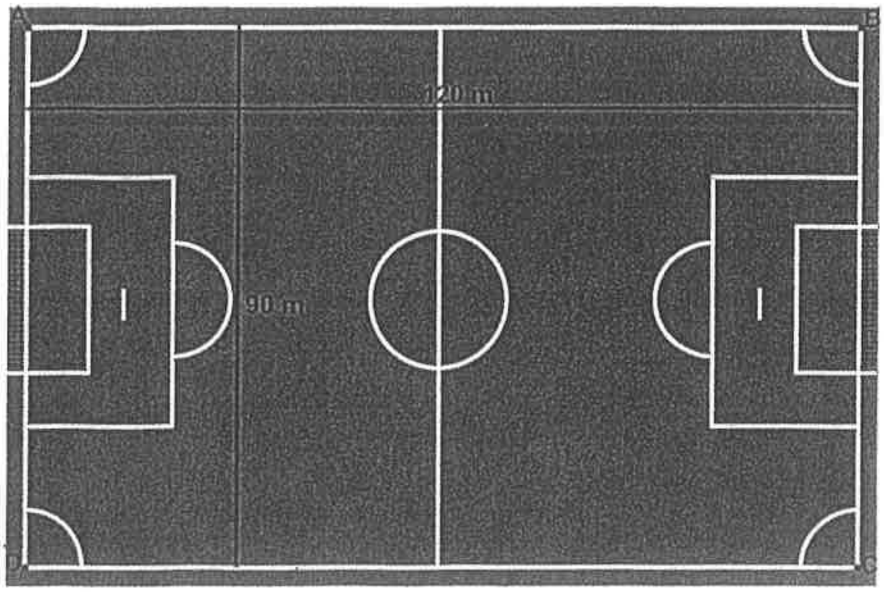


9. Alan wants to know the height of a tree in his back garden. Standing 5 m from the foot of the tree, and using a clinometer, he measures the angle of elevation of the top of the tree to be 33°. If Alan is 170 cm in height, find the height of the tree.



10. A flagpole AB, with A at the top, is held upright by two ropes, AC and AD, fixed on horizontal ground at C and D. $|AC| = 10$. If $|\angle ACB| = 25°$ and $|\angle ADB| = 36°$, find $|BD|$ correct to one decimal place.

11. A football pitch $ABCD$ is shown. During a training session, a coach asks each player to run from A to C and back to A. The pitch is 120 m long and 90 m wide. Calculate the distance covered by each player during this exercise.



11.6 DEGREES, MINUTES AND SECONDS (DMS)

Higher Level

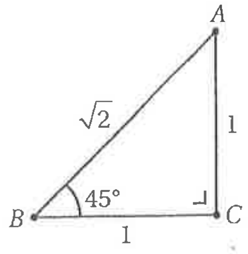
In our course, we measure angles in units called degrees. One degree is $\frac{1}{360}$ of the angle in a full circle. A degree can be subdivided into smaller units known as minutes. There are 60 minutes in one degree. We write $60' = 1°$.

Therefore, if we wish to change degrees to minutes. We multiply the number of degrees by 60.

A minute can be subdivided into 60 seconds, but these are so small that we do not bother with them.

2. 45°

Consider the isosceles right angled triangle ABC with $|\angle ACB| = 90°$, $|AC| = 1$ and $|BC| = 1$. Then $|\angle ABC| = 45° = |\angle BAC|$, as the base angles of an isosceles triangle are equal, and in this case add up to 90°. Also, by Pythagoras' Theorem, $|AB| = \sqrt{2}$.



Thus: $\cos 45° = \frac{1}{\sqrt{2}}$, $\sin 45° = \frac{1}{\sqrt{2}}$, $\tan 45° = 1$.

In summary:

A	$\frac{\pi}{3}$ (60°)	$\frac{\pi}{4}$ (45°)	$\frac{\pi}{6}$ (30°)
$\cos A$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\sin A$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan A$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$

Exercises 14.2

Use your calculator to evaluate each of the following, correct to four decimal places.

1. cos 18° **2.** sin 56° **3.** tan 41° **4.** sin 79°

5. sin 24°30' **6.** cos 28°22' **7.** tan 62°47' **8.** tan 11°48'

Use your calculator to evaluate the angle A, correct to two decimal places.

9. $\sin A = 0{\cdot}75$ **10.** $\tan A = 1{\cdot}32$ **11.** $\cos A = 0{\cdot}68$ **12.** $\sin A = 0{\cdot}8$

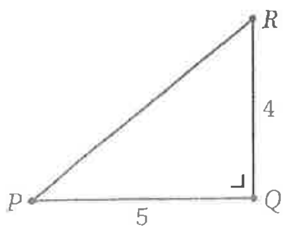
13. $\tan A = 0{\cdot}3672$ **14.** $\cos A = 0{\cdot}7845$ **15.** $\sin A = 0{\cdot}8432$ **16.** $\cos A = 0{\cdot}1976$

17. In the triangle ABC, $|\angle ACB| = 90°$, $|\angle BAC| = 41°$ and $|BC| = 8$ cm.
(i) Calculate $|AB|$, correct to two decimal places.
(ii) Calculate $|AC|$, correct to two decimal places.

18. In the triangle PMN, $|\angle PMN| = 90°$, $|\angle MPN| = 28°$ and $|PN| = 7$ cm.
(i) Calculate $|PM|$, correct to two decimal places.
(ii) Calculate $|MN|$, correct to two decimal places.

19. In the triangle ABC, $|\angle ABC| = 90°$, $|\angle BAC| = 52°$ and $|BC| = 11$ cm.
(i) Calculate $|AB|$, correct to two decimal places.
(ii) Calculate $|AC|$, correct to two decimal places.

20. In the triangle PQR, $|PQ| = 5$, $|QR| = 4$ and $|\angle PQR| = 90°$. Calculate
(i) $|\angle RPQ|$, correct to one decimal place,
(ii) $|PR|$, in the form $\sqrt{k}$, where $k \in \mathbb{N}$.



21. In the triangle PQR, $|\angle PRQ| = 90°$, $|PR| = 11{\cdot}3$ cm and $|QR| = 9{\cdot}1$ cm.
(i) Calculate $|\angle QPR|$, correct to the nearest degree.
(ii) Calculate $|PQ|$, correct to one decimal place.

22. In the triangle ABC, $|\angle ABC| = 90°$, $|AB| = \sqrt{13}$ cm and $|BC| = \sqrt{7}$ cm.
(i) Calculate $|\angle BAC|$, correct to the nearest degree.
(ii) Calculate $|AC|$, expressing your answer in the form $\sqrt{k}$, where $k \in \mathbb{N}$.

23. If $\cos A = \frac{2}{5}$, for $0° < A < 90°$, express $\sin A$ in the form $\frac{\sqrt{k}}{m}$, where $k, m \in \mathbb{N}$.

24. If $\sin A = \frac{\sqrt{7}}{4}$, for $0° < A < 90°$, express $\cos A$ in the form $\frac{k}{m}$, where $k, m \in \mathbb{N}$.

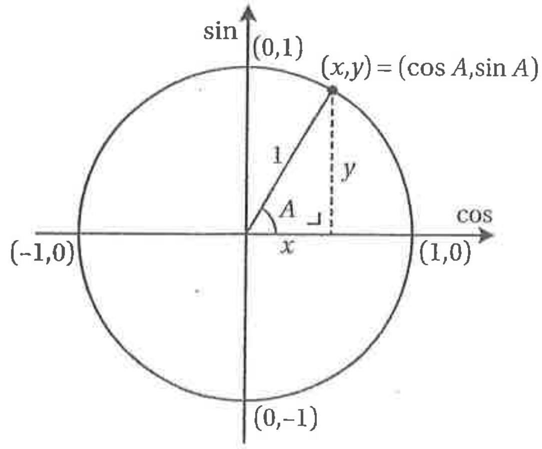
25. If $\tan A = \frac{m}{n}$, for $0° < A < 90°$, express $\sin A$ in terms of $m, n \in \mathbb{N}$.

26. If $\sin A = \frac{t}{t+1}$, for $0° < A < 90°$, express $\tan A$ in terms of $t \in \mathbb{N}$.

14.3 Trigonometric Functions of All Angles

The right-angled triangle definitions of the trig ratios only hold for angles between 0° and 90°. But we would like sine, cosine and tan to be defined for all angles, or at least as many angles as possible. This requires a new definition, but one which is consistent with the definition of the trig ratios for angles between 0° and 90°. A long time ago, the definition was extended so that the likes of cos 147° could have meaning.

The **trigonometric functions**, $\cos A$ and $\sin A$, are defined for all angles A using 'the unit circle'. This is the circle with centre (0,0) and radius length 1. Starting from the positive side of the x-axis, anticlockwise angles are taken as positive and clockwise angles are taken as negative.



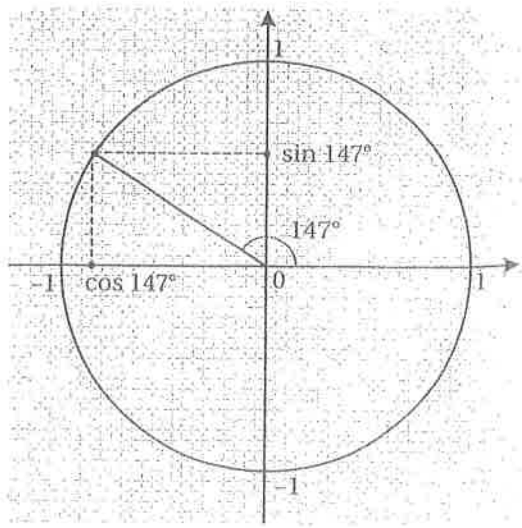
Consider the triangle shown opposite and the point (x,y) on the unit circle. From the trig ratios,

$$\cos A = \frac{x}{1} = x \qquad \text{and} \qquad \sin A = \frac{y}{1} = y.$$

Thus, $(x,y) = (\cos A, \sin A)$.

It is natural to extend this definition to all angles $A \in \mathbb{R}$.

Thus, in theory, to find cos 147° we rotate from the positive side of the x-axis, through 147° in an anticlockwise direction. The point on the unit circle at the end of the radius has coordinates (cos 147°, sin 147°).



A calculator gives cos 147° as being equal to −0·8387, which looks reasonable from the diagram. Likewise, from a calculator, sin 147° = 0·5446, which again agrees with what we would expect from the diagram.

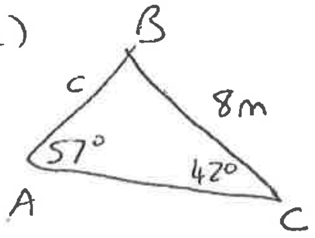
$\tan A$ is defined in terms of $\cos A$ and $\sin A$, just as it was for acute angles, i.e. $\tan A = \frac{\sin A}{\cos A}$. Thus, to estimate tan 147° from the graph, we could write

$$\tan 147° = \frac{\sin 147°}{\cos 147°} = \frac{0{\cdot}5446}{-0{\cdot}8387} = -0{\cdot}6494.$$

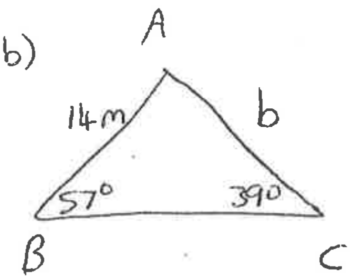
Exercise H :

1. Find the side marked with a letter, to one decimal place :

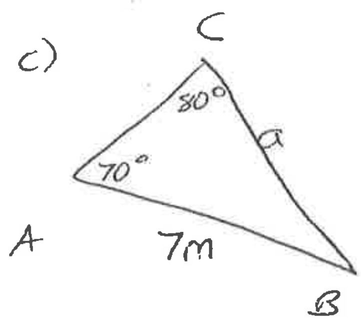
a)



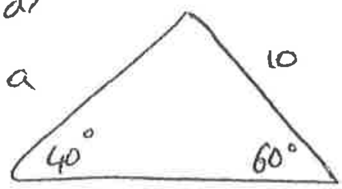
b)



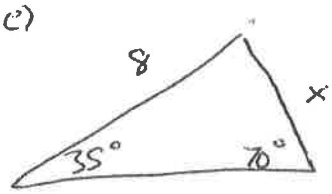
c)



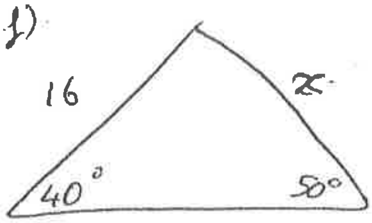
d)



e)

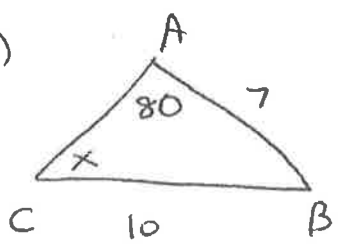


f)

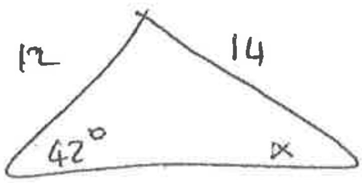


2. Find angle x, to the nearest degree :

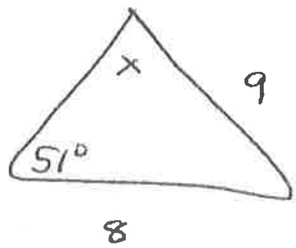
a)



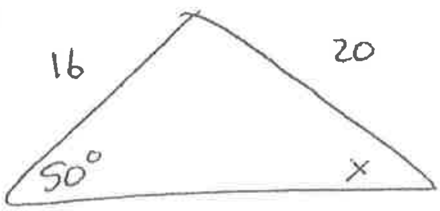
b)



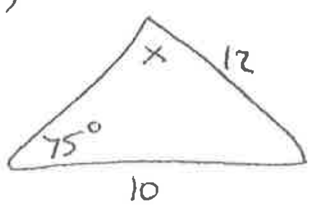
c)



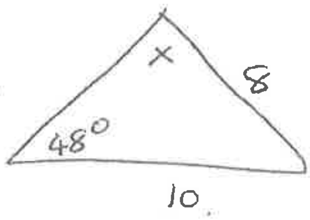
d)



e)

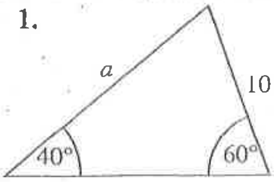


f)

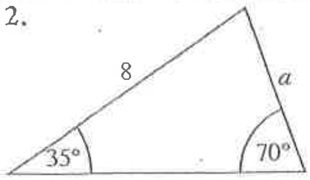


Sine Rule

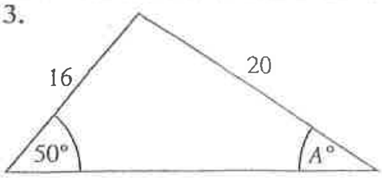
1.



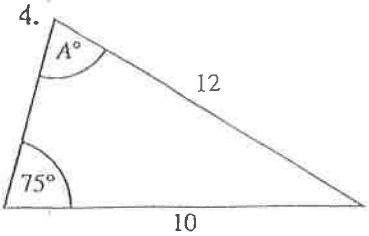
2.



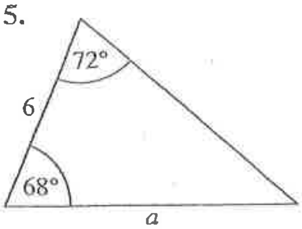
3.



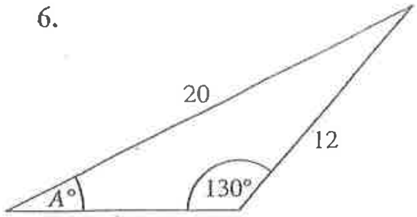
4.



5.



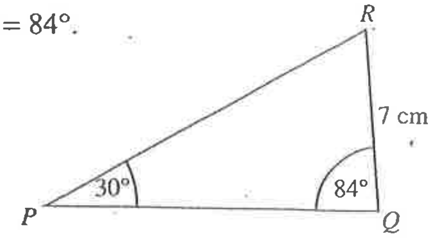
6.



7. In triangle PQR, $|QR| = 7$ cm, $|\angle QPR| = 30°$ and $|\angle PQR| = 84°$.
Calculate:

(i) $|PR|$ correct to the nearest integer

(ii) $|PQ|$ correct to the nearest integer



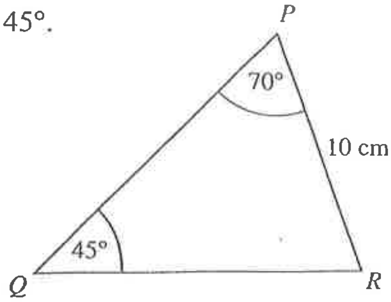
8. In triangle PQR, $|PR| = 10$ cm, $|\angle QPR| = 70°$ and $|\angle PQR| = 45°$.

(i) Find $|\angle PRQ|$.

Calculate the following, correct to one decimal place.

(ii) $|QR|$

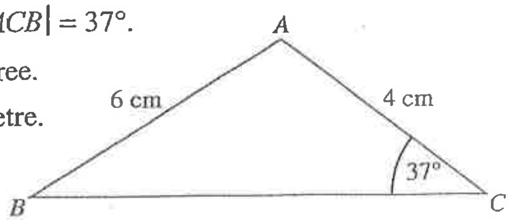
(iii) $|PQ|$



9. In triangle ABC, $|AC| = 4$ cm, $|AB| = 6$ cm and $|\angle ACB| = 37°$.

(i) Calculate $|\angle ABC|$, correct to the nearest degree.

(ii) Calculate $|BC|$, correct to the nearest centimetre.



10. In the diagram, $PQ \perp QR$, $|PQ| = 8$ m and $|QR| = 15$ m.

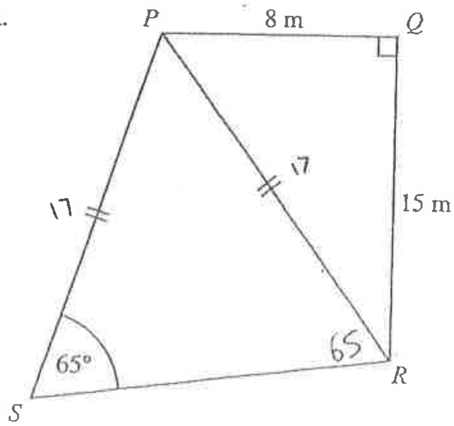
(i) Find $|PR|$.

Given $|PS| = |PR|$ and $|\angle PSR| = 65°$:

(ii) Find $|\angle SPR|$

(iii) Find the area of triangle PRS, correct to the nearest m^2

(iv) Find $|SR|$, correct to two decimal places

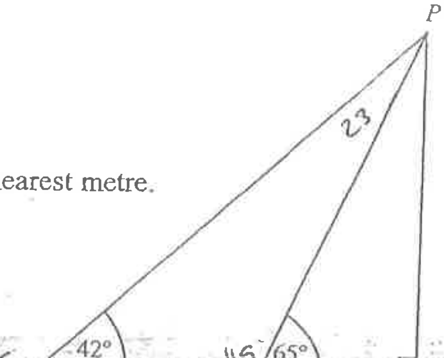


11. In the diagram, $PQ \perp SQ$, $|SR| = 60$ m, $|\angle PSR| = 42°$ and $|\angle PRQ| = 65°$. Calculate:

(i) $|\angle SPR|$

(ii) $|PR|$, correct to the nearest metre

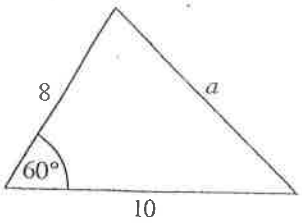
(iii) Hence or otherwise, calculate $|PQ|$ correct to the nearest metre.



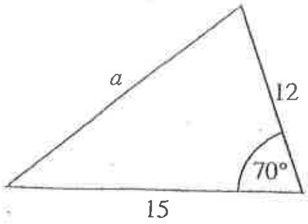
Cosine Rule

In questions 1–6, use the cosine rule to calculate the following in the triangles below: (i) a, correct to two decimal places or (ii) A, correct to the nearest degree.

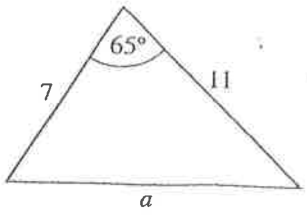
1.



2.



3.



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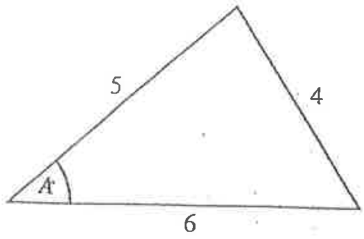
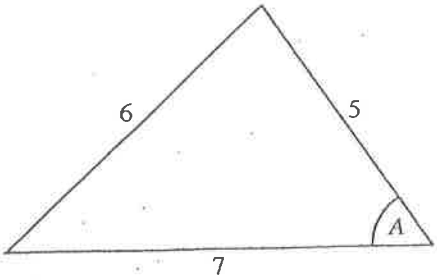
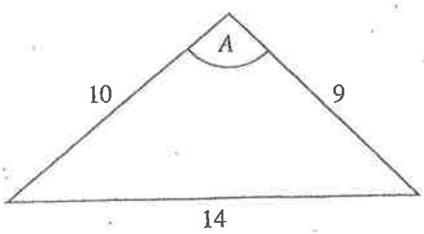


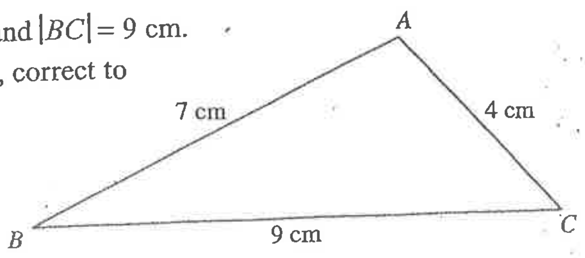
5.



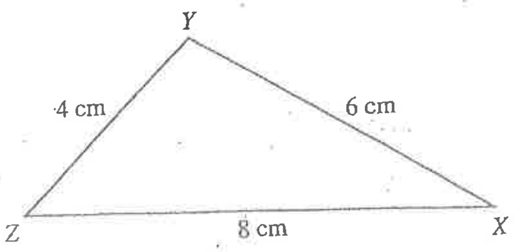
6.



7. In triangle ABC, $|AB| = 7$ cm, $|AC| = 4$ cm and $|BC| = 9$ cm. Calculate the measure of the greatest angle, correct to one decimal place.



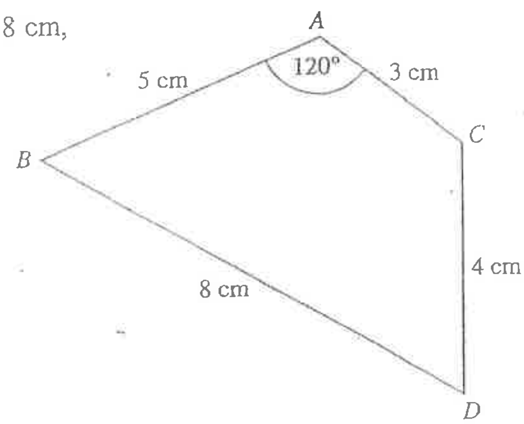
8. In triangle XYZ, $|XY| = 6$ cm, $|XZ| = 8$ cm and $|YZ| = 4$ cm. Calculate the measure of the smallest angle, correct to the nearest degree.



9. In the diagram, $|AB| = 5$ cm, $|AC| = 3$ cm, $|BD| = 8$ cm, $|CD| = 4$ cm and $|\angle BAC| = 120°$.

(i) Calculate $|BC|$.

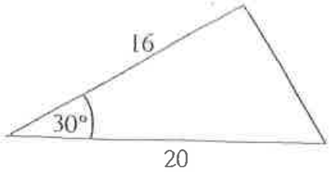
(ii) Find the measure of $|\angle BDC|$, correct to the nearest degree.



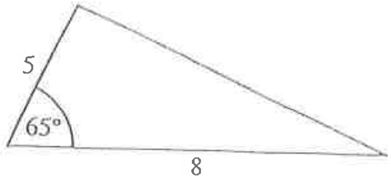
Area

In questions 1–9, find, correct to two decimal places, the area of each of the following triangles, where the lengths of the sides are in centimetres.

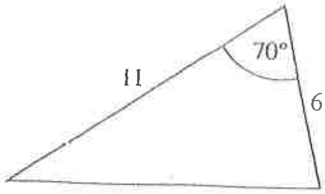
1.



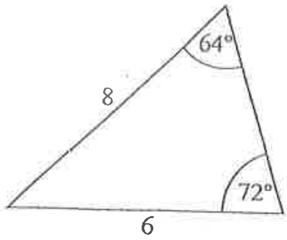
2.



3.



4.



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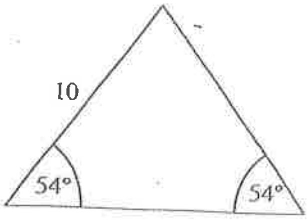

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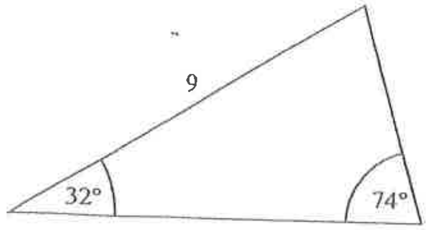

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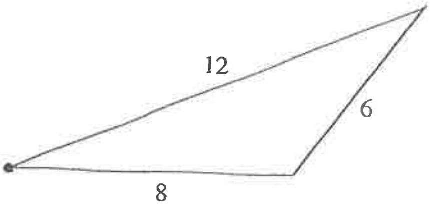

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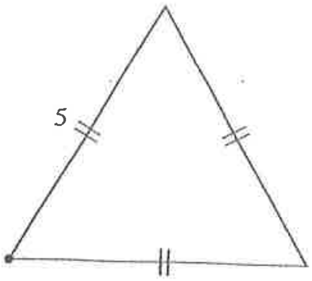

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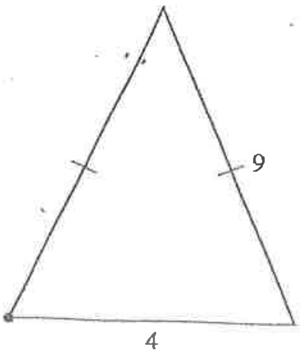
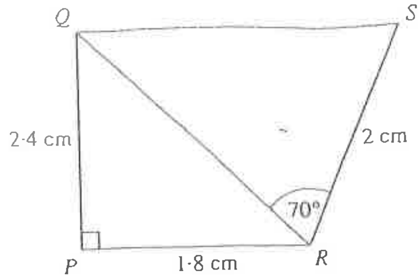


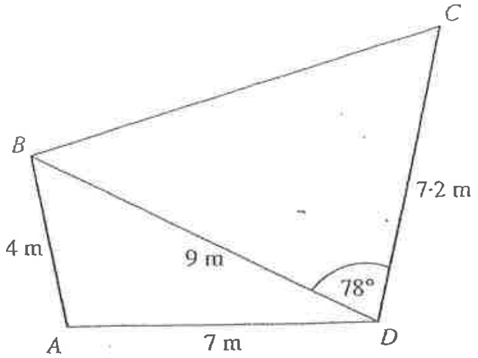
For questions 10–14, a rough diagram may help. Where relevant, give all answers correct to two decimal places.

10. In triangle PQR, $|PR| = 8$ m, $|PQ| = 7$ m and $|\angle QPR| = 54°$. Calculate the area of triangle PQR.
11. In triangle ABC, $|BC| = 8$ cm, $|AC| = 10$ cm and $|\angle ABC| = 48°$. Calculate the area of triangle ABC.
12. In triangle XYZ, $|XY| = 14$ cm, $|XZ| = 9$ cm and $|YZ| = 11$ cm. Calculate the area of triangle XYZ.
13. In triangle PQR, $|QR| = 6$ m, $|PR| = 16$ m, $|\angle RPQ| = 40°$ and $|\angle PQR| = 30°$. Calculate the area of triangle PQR.
14. In triangle ABC, $|AC| = 14$ cm, $|\angle ABC| = 70°$ and $|\angle BAC| = 40°$. Calculate the area of triangle ABC.

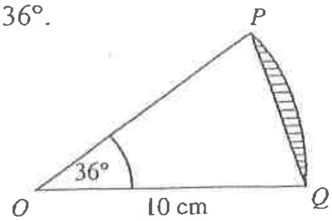
15. The diagram shows the quadrilateral $PQSR$.
$QP \perp PR$, $|PQ| = 2{\cdot}4$ cm, $|PR| = 1{\cdot}8$ cm, $|RS| = 2$ cm and $|\angle QRS| = 70°$. Calculate:
(i) $|QR|$
(ii) The area of triangle PQR
(iii) The area of $PQSR$, correct to two decimal places



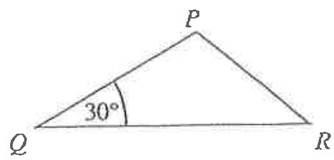
16. The diagram shows the quadrilateral $ABCD$.
$|AD| = 7$ m, $|AB| = 4$ m, $|BD| = 9$ m, $|CD| = 7{\cdot}2$ m and $|\angle BDC| = 78°$. Calculate:
(i) The area of triangle ABD, correct to one decimal place
(ii) The area of $ABCD$, correct to one decimal place



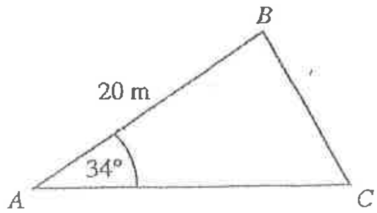
17. OPQ is a sector of a circle with a radius of 10 cm and $|\angle POQ| = 36°$.
(i) Taking $\pi = 3{\cdot}14$, calculate the area of the sector OPQ.
(ii) Calculate, correct to one decimal place:
(a) The area of triangle OPQ
(b) The area of the shaded segment



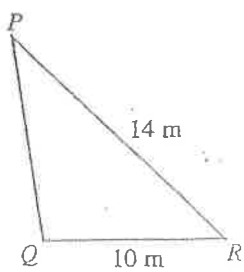
18. In triangle PQR, $|PQ| = 8$ cm and $|\angle PQR| = 30°$.
If the area of triangle $PQR = 48$ cm^2, calculate $|QR|$.



19. **(i)** Calculate sin 34° correct to two decimal places.
(ii) In triangle ABC, $|AB| = 20$ m and $|\angle BAC| = 34°$. If the area of triangle $ABC = 145{\cdot}6$ m^2, find $|AC|$, using the value of sin 34° obtained in part **(i)**.

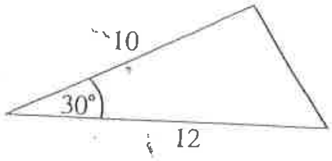


20. In triangle PQR, $|PR| = 14$ m and $|QR| = 10$ m. If the area of triangle PQR is 45 m^2, calculate $|\angle PRQ|$, correct to the nearest degree.

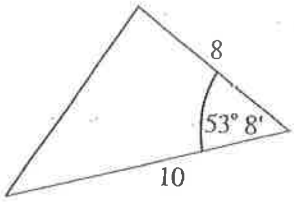


1. Find the areas of these triangles, to the nearest square unit:

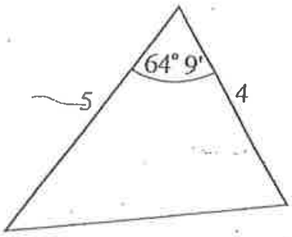
(i)



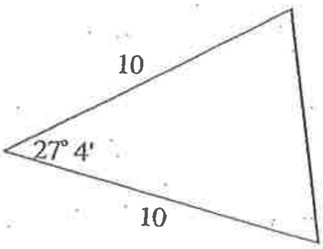
(ii)



(iii)



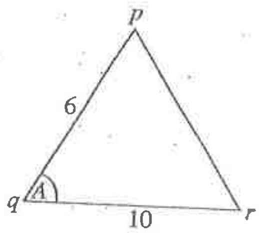
(iv)



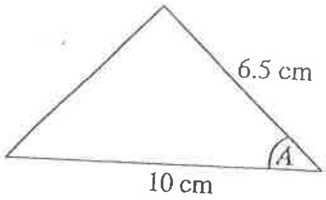
2. If $\cos A = 0.6$, find

(i) Sine A,

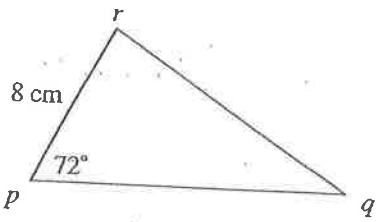
(ii) area of Δpqr.



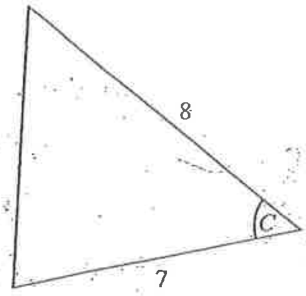
3. (a) If the area of this triangle is 25 cm², find the measure of the acute angle A, to the nearest degree.



(b) If the area of Δpqr is 40 cm², find $|pq|$ to the nearest millimetre.

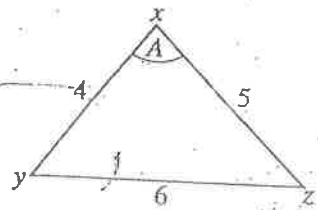


(c) If the area of this triangle is 14 square units, find the angle C:

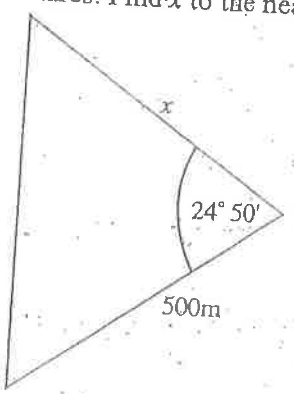


4. (i) Find A to the nearest degree.

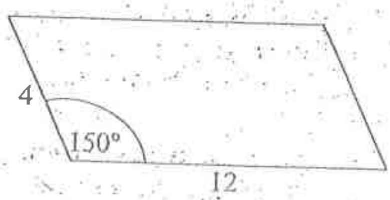
(ii) Find the area of Δxyz.



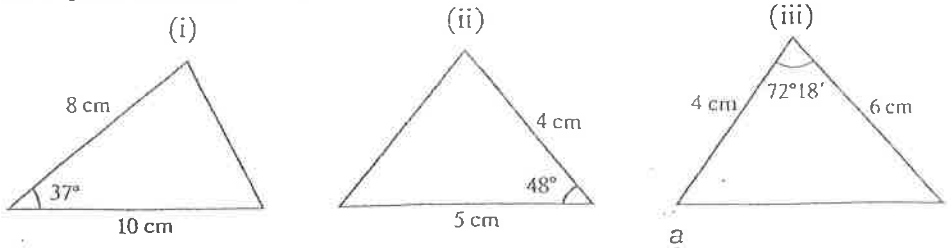
5. The area of this triangular field is 4.2 hectares. Find x to the nearest metre.



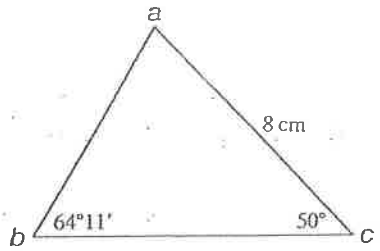
6. Find the area of this parallelogram:



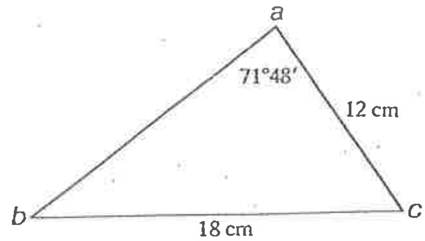
1. Find the area of each of the triangles shown below.
Give your answers in cm^2, correct to one decimal place.



2. In the given triangle $|ac| = 8$ cm, $|\angle abc| = 64°11'$ and $|\angle acb| = 50°$.
Find (i) $|ab|$
(ii) the area of Δabc
Give each answer correct to one decimal place.

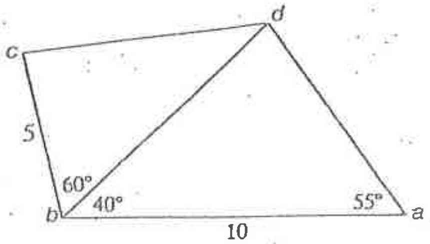


3. In the given triangle $A = 71°48'$, $|bc| = 18$ cm and $|ac| = 12$ cm.
Find (i) $|\angle abc|$
(ii) $|ab|$, correct to the nearest cm.

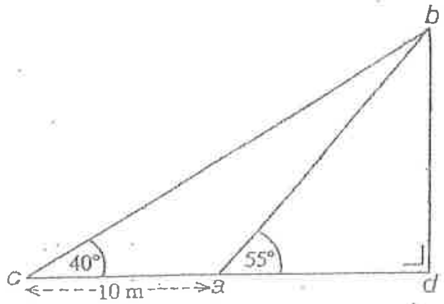


4. In a triangle abc, $a = 5$, $b = 6$ and $A = 54°6'$.
Find (i) $|\angle abc|$, correct to the nearest degree
(ii) the length of $[ab]$, correct to 1 decimal place.

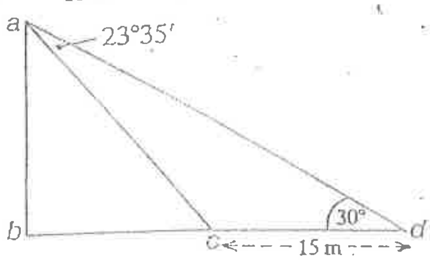
5. In the given quadrilateral $abcd$, $|ab| = 10$, $|bc| = 5$ $|\angle abd| = 40°$, $|\angle dab| = 55°$ and $|\angle dbc| = 60°$.
Find (i) $|bd|$
(ii) the area of the quadrilateral $abcd$.
Give each answer correct to 1 decimal place.



6. In the given triangle $|ac| = 10$ m, $|\angle acb| = 40°$, $|\angle bad| = 55°$ and $bd \perp cd$.
Find (i) $|bc|$ and (ii) $|bd|$, giving each answer correct to the nearest metre.

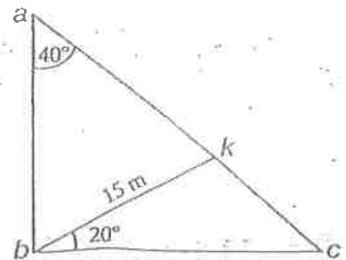


7. In the given triangle $|cd| = 15$ m, $|\angle adc| = 30°$ and $|\angle cad| = 23°35'$.
Find (i) $|ac|$ and (ii) $|ab|$, in metres, correct to 1 decimal place.

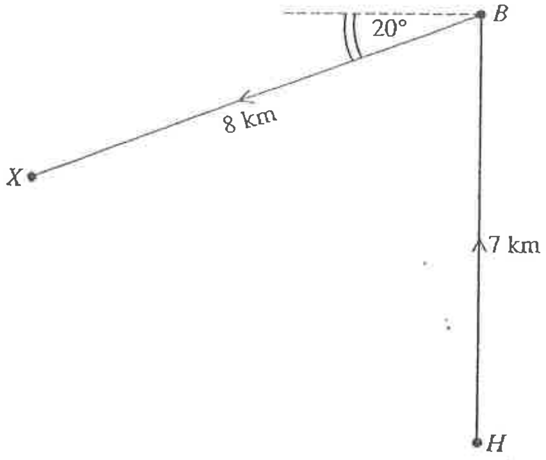


8. In the given diagram $ab \perp bc$, $|bk| = 15$ m, $|\angle cbk| = 20°$ and $|\angle bac| = 40°$.

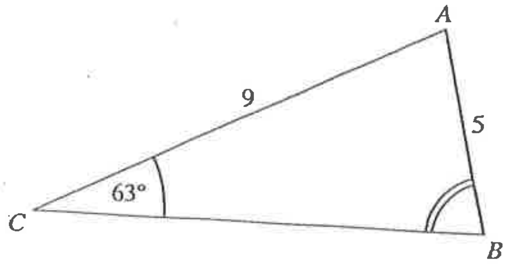
Calculate $|ak|$, in metres, correct to one decimal place.



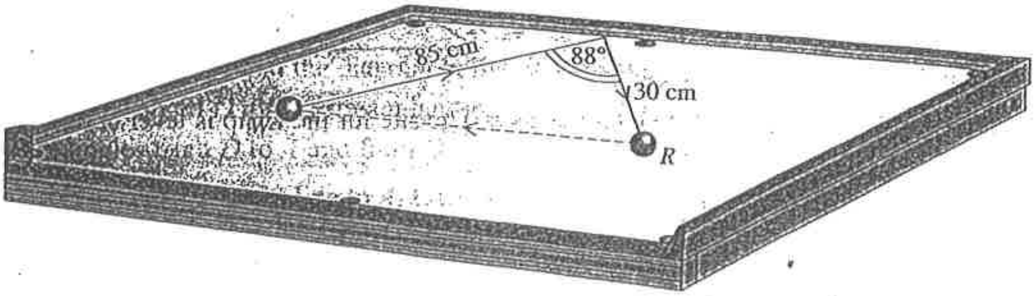
1. A boat sets sail from a harbour, H, and travels 7 km due north to a marker buoy, B. At B the boat turns W 20° S and travels a further 8 km before stopping at X. Calculate the straight line distance from H to X, correct to two decimal places.



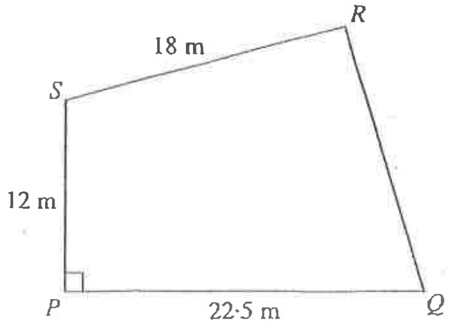
2. Use the sine rule to show that triangle ABC is an impossible triangle.



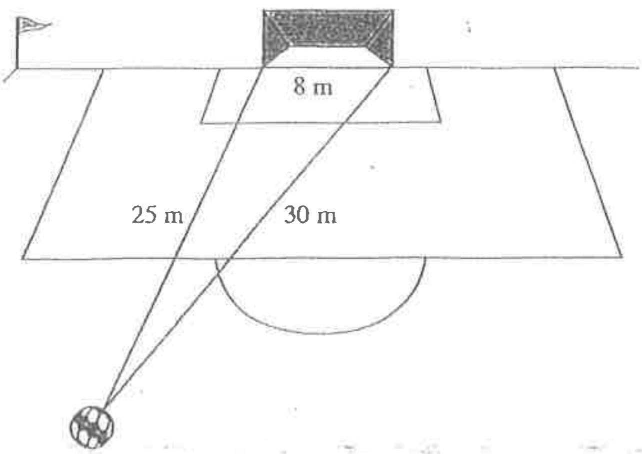
3. A snooker player cues the white (W) ball onto the cushion to rebound and hit the red (R) ball, as shown in the diagram. The white ball travels 85 cm before being deflected 88° by the cushion. It then travels 30 cm before hitting the red ball. The white ball then returns in a straight line to its original position. Find the total distance travelled by the white ball, correct to the nearest centimetre.



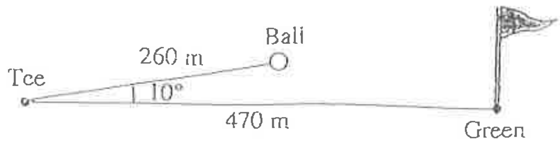
4. A garden, $PQRS$, is in the shape of a quadrilateral with $SP \perp PQ$, $|PQ| = 22{\cdot}5$ m, $|PS| = 12$ m, $|SR| = 18$ m and $|\angle QSR| = 42°$. Calculate the following.
   (i) $|QS|$
   (ii) $|RQ|$, correct to the nearest metre
   (iii) The area of the garden correct to the nearest m^2



5. The goal posts on a soccer field are 8 m apart. A player kicks for a goal when he is 30 m from one post and 25 m from the other.
   Find the angle opposite the goal line, measured from both goal posts to where the ball is positioned, correct to the nearest degree.

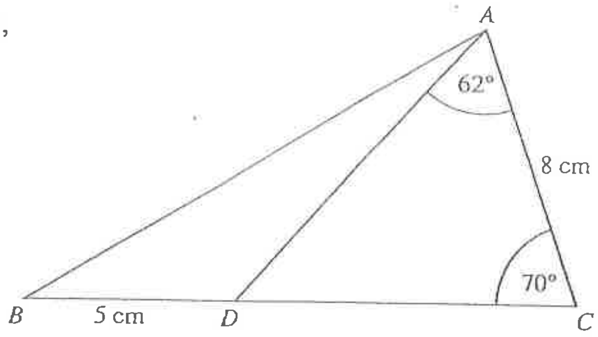


6. The third hole on a golf course is 470 m from the tee. A ball is driven from the tee a distance of 260 m. However, the drive is 10° off the line to the hole, as shown.

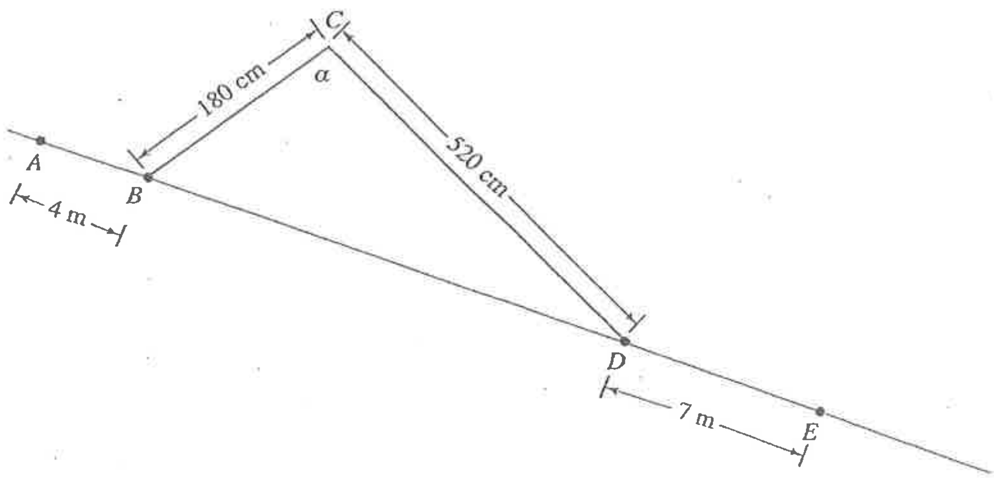


How far from the hole is the ball, correct to the nearest m?

7. ABC is a triangle and D is a point on $[BC]$, as shown. $|BD| = 5$ cm, $|AC| = 8$ cm, $|\angle ACD| = 70°$ and $|\angle DAC| = 62°$.
Find the following correct to one decimal place.
(i) $|DC|$
(ii) The area of triangle ABC

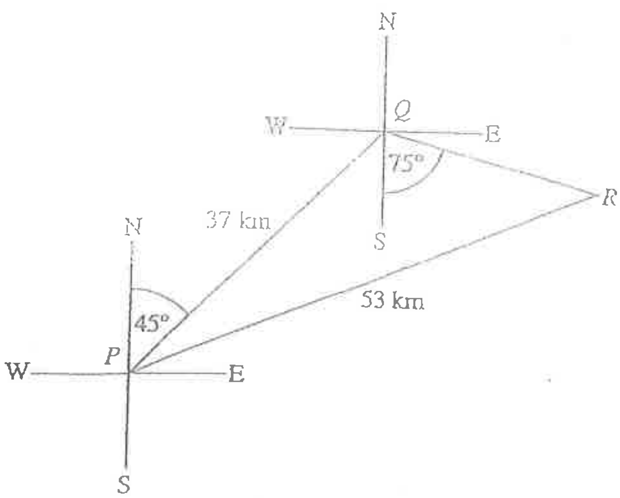


8. Nina is an engineer. She is asked to design a ramp for mountain bike enthusiasts. The site for the ramp is level but sloped, as in the diagram.



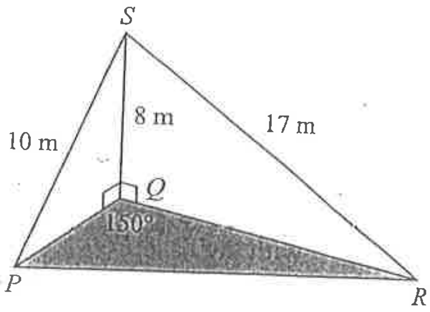
For health and safety reasons, $|AB| = 4$ m, $|DE| = 7$ m and $\alpha \geq 96°$.
For the ramp to meet the bikers' specifications, $\alpha \leq 108°$, $|BC| = 180$ cm and $|CD| = 520$ cm.
In order for Nina to meet the requirements, find:
(i) The maximum $|AE|$ correct to the nearest centimetre
(ii) The minimum $|AE|$ correct to the nearest centimetre
(iii) Comment on the difference between the maximum and minimum $|AE|$. Would Nina favour case (i) over case (ii)? Justify your answer.

9. A ship, Q, is 37 km from a port, P. The direction of Q from P is N 45° E. A second ship, R, is 53 km from P. The direction of R from Q is S 75° E.
Calculate:
(i) $|\angle QRP|$, correct to one decimal place
(ii) $|QR|$, correct to two decimal places

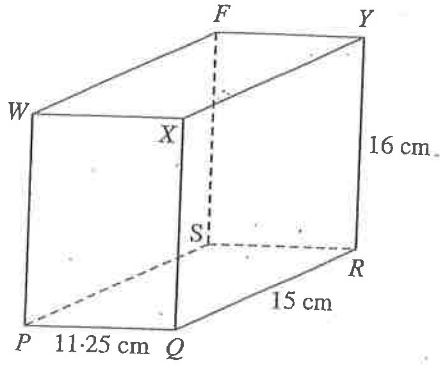


Exercise 9.8

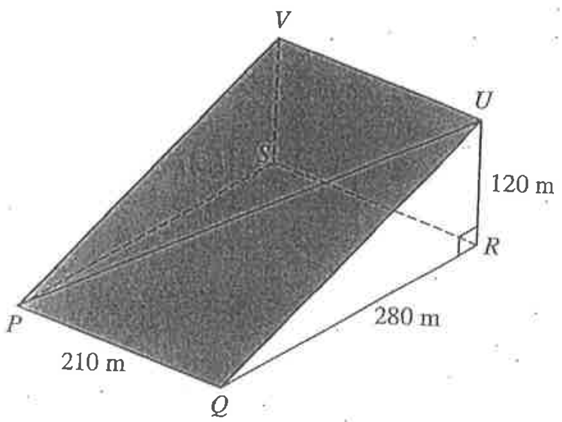
1. P, Q and R are three points on level ground. $[QS]$ represents a vertical pole of height 8 m. $|\angle PQR| = 150°$, $|PS| = 10$ m and $|RS| = 17$ m. Find:
   (i) $|PQ|$ **(ii)** $|QR|$ **(iii)** Area of triangle PQR
   (iv) $|PR|$, correct to two decimal places



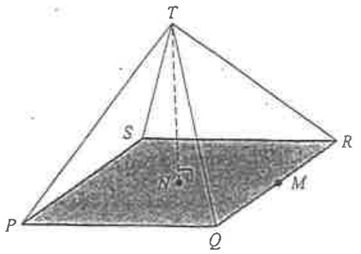
2. The diagram shows a cuboid with dimensions as shown. Calculate:
   (i) $|PR|$
   (ii) $|\angle RPY|$, correct to one decimal place



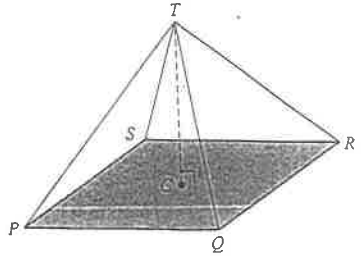
3. On the diagram, the rectangle $PQUV$ represents an artificial ski slope. Rectangles $PQRS$ and $RSVU$ are at right angles to each other. Jack skis in a straight line from U to P at an average speed of 5 m per second.
   (i) How long, in seconds, does it take Jack to ski from U to P?
   (ii) A health and safety expert says that the maximum slope of an artificial ski slope must be 25°. Would this artificial ski slope be passed as safe by this expert? Justify your answer.
   (iii) Calculate, correct to one decimal place, the difference in the angle between skiing straight down the slope and skiing from U to P.



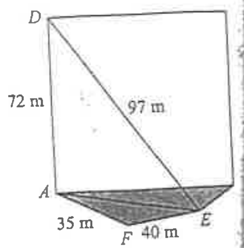
4. $PQRST$ is a square-based pyramid of side 12 cm. The vertex T is directly above N, the centre of the square $PQRS$.
M is the midpoint of $[QR]$ and $|TM| = 10$ cm.
(i) Find $|TN|$.
(ii) Calculate, correct to two decimal places, the angle the face QRT makes with the base $PQRS$.



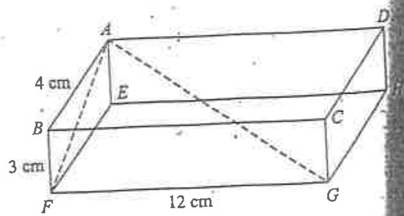
5. $PQRST$ is a square-based pyramid of side 6 cm. The vertex T is directly above C, the centre of the square $PQRS$. $|TC| = 4$ cm.
Calculate:
(i) $|\angle TQR|$, to the nearest degree
(ii) The total surface area of the pyramid



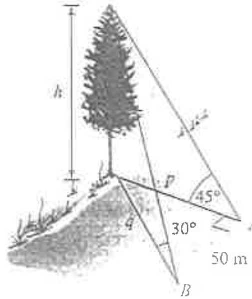
6. A, F and E are points on horizontal ground. D is a point on a vertical wall directly above A. $|AD| = 72$ m, $|DE| = 97$ m, $|AF| = 35$ m and $|FE| = 40$ m.
(i) Calculate $|AE|$.
(ii) Hence, calculate $|\angle AFE|$.



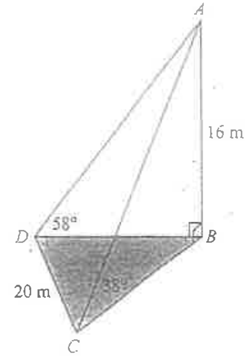
7. The diagram shows a rectangular box.
Rectangle $ABCD$ is the top of the box and rectangle $EFGH$ is the base of the box.
$|AB| = 4$ cm, $|BF| = 3$ cm and $|FG| = 12$ cm.
(i) Find $|AF|$.
(ii) Find $|AG|$.
(iii) Find the measure of the acute angle between $[AG]$ and $[DF]$. Give your answer correct to the nearest degree.



8. The diagram shows a river with parallel banks p m apart. A vertical tree, of height h m, is directly opposite the point A, as shown. A woman wants to find the height of the tree. From A the angle of elevation of the top of the tree is 45°. She then walks to a point B, which is 50 m downstream, as shown. The distance from B to the base of the tree is q m and the angle of elevation of the top of the tree from B is 30°.
(i) Express p and q in terms of h.
(ii) Write a quadratic equation involving p and q.
(iii) Calculate h and write your answer in the form $25\sqrt{k}$.



9. The diagram shows a tetrahedron $ABCD$, where $|AB| = 16$ m, $|DC| = 20$ m, $|\angle ACB| = 38°$ and $|\angle ADB| = 58°$. $[AB]$ is vertical and points B, C and D are on level ground. Find, correct to two decimal places:
(i) $|AC|$
(ii) $|AD|$
(iii) $|\angle DAC|$



10. A vertical radio mast $[PQ]$ stands on flat horizontal ground. It is supported by three cables that join the top of the mast, Q, to the points A, B and C on the ground. The foot of the mast, P, lies inside the triangle ABC. Each cable is 52 m long and the mast is 48 m high.
(i) Find the (common) distance from P to each of the points A, B and C.
(ii) Given that $|AC| = 38$ m and $|AB| = 34$ m, find $|BC|$, correct to one decimal place.